# The Safe Withdrawal Rate: Evidence from a Broad Sample of Developed Markets

Aizhan Anarkulova<sup>\*</sup> S

Scott Cederburg<sup>†</sup>

<sup>†</sup> Michael S. O'Doherty<sup>‡</sup>

Richard Sias<sup>§</sup>

July 18,  $2023^{\P}$ 

#### Abstract

We use a comprehensive new dataset of asset-class returns in 38 developed countries to examine a popular class of retirement spending rules that prescribe annual withdrawals as a constant percentage of the retirement account balance. A 65-year-old couple willing to bear a 5% chance of financial ruin can withdraw just 2.26% per year, a rate materially lower than conventional advice (e.g., the 4% rule). Our estimates of failure rates under conventional withdrawal policies have important implications for individuals (e.g., savings rates, retirement timing, and retirement consumption), public policy (e.g., participation rates in means-tested programs), and society (e.g., elderly poverty rates).

JEL classifications: C15, D31, G10, G11, G12, G15, G17, G51, N20 Key words: Retirement, safe withdrawal rate, survivor bias, easy data bias, financial ruin

<sup>\*</sup>Goizueta Business School, Emory University. Email: aizhan.anarkulova@emory.edu.

<sup>&</sup>lt;sup>†</sup>Eller College of Management, University of Arizona. Email: cederburg@arizona.edu.

<sup>&</sup>lt;sup>‡</sup>Trulaske College of Business, University of Missouri. Email: odohertym@missouri.edu.

<sup>&</sup>lt;sup>§</sup>Eller College of Management, University of Arizona. Email: sias@arizona.edu.

<sup>&</sup>lt;sup>¶</sup>We thank Benjamin Felix, Ashish Tiwari, and Harry Turtle for helpful comments and suggestions. This research was supported by a grant from the Robert J. Trulaske, Sr. College of Business Large Grant Program.

# 1 Introduction

An aging population, greater longevity, earlier retirements, and a shift from defined benefit to defined contribution retirement plans are driving a renewed focus on determining how retirees should spend down their savings.<sup>1</sup> The most popular strategy calls for a constant real withdrawal of 4% each year of the household's financial wealth at retirement.<sup>2</sup> That is, a household with \$1 million in savings upon retirement makes a \$40,000 withdrawal in the first year and inflationadjusted withdrawals of \$40,000 in subsequent years. The 4% rule originates from Bengen (1994), who finds that a retirement strategy with 50% in stocks, 50% in government bonds, and a 4%inflation-adjusted withdrawal rate survives each 30-year period in the US historical record from 1926 to 1991.<sup>3</sup> The "safe" 4% spending rule is ubiquitous and recommended by financial advisors, brokerages, mutual fund companies, retirement groups, and the popular press. Choi (2022) reviews the 50 most popular personal finance books and finds that, of the 12 providing explicit retirement spending advice, seven recommend a 4% spending rule and four recommend an even higher rate (5% to 8%). The 4% rule appears in congressional testimony and is detailed on multiple state and federal government websites.<sup>4</sup> Its ubiquity is also apparent in the growing Financial Independence Retire Early (FIRE) movement, wherein many millennials (22% according to a recent Vanguard survey) are planning for early retirement based on the 4% rule.<sup>5</sup>

The 4% rule is a leading example of the divergence between finance theory and practice. Normative portfolio choice models [e.g., Yaari (1965) and Davidoff, Brown, and Diamond (2005)] prescribe full or considerable annuitization of assets at the onset of retirement to address the risk of households outliving their wealth. In practice, however, few retirees purchase life annuities [Poterba, Venti, and Wise (2011)], and annuitization has, if anything, declined in popularity [Brown, Poterba, and Richardson (2023)]. Academic explanations for the annuitization puzzle include adverse selection and the actuarially unfair pricing of annuity contracts [e.g., Mitchell, Poterba, Warshawsky, and Brown (1999)], bequest motives [e.g., Lockwood (2018)], health risks [e.g., Reichling and Smetters

<sup>&</sup>lt;sup>1</sup>See Benartzi, Previtero, and Thaler (2011) and Poterba (2014) for evidence of population aging, increased longevity, shifts to earlier retirement, and migration to defined contribution plans.

<sup>&</sup>lt;sup>2</sup>For a practitioner summary of popular retirement income strategies, see https://www.forbes.com/sites/ jamiehopkins/2019/04/16/comparing-the-3-most-popular-retirement-income-strategies/?sh=9a5b3b9166e5.

<sup>&</sup>lt;sup>3</sup>Subsequent studies consider alternative horizons or stock-bond allocations [e.g., Cooley, Hubbard, and Walz (1998)], international diversification [e.g., Cooley, Hubbard, and Walz (2003)], and simulations based on parameterized distributions [e.g., Pye (2000) and Benz, Ptak, and Rekenthaler (2021)] and reach largely the same conclusion.

<sup>&</sup>lt;sup>4</sup>See Scott, Sharpe, and Watson (2009) for examples of references to the 4% rule. See https://www.aging.senate. gov/imo/media/doc/hr158su.pdf for congressional testimony.

<sup>&</sup>lt;sup>5</sup>See https://www.campfirefinance.com/4-percent-rule/ for an overview of the 4% rule and its link to the FIRE movement, and see https://corporate.vanguard.com/content/dam/corp/research/pdf/Fuel-for-the-F. I.R.E.-Updating-the-4-rule-for-early-retirees-US-ISGFIRE\_062021\_Online.pdf for survey evidence.

(2015)], and behavioral factors.<sup>6</sup> Thus, there is debate regarding the extent to which retirees should annuitize versus self-fund retirement.

Current retirement spending practices demonstrate a revealed preference for spending rules over annuitization, such that the efficacy of spending rules is an important issue. Recent reviews by Choi (2022) and Cochrane (2022) emphasize the need to better understand the gap between theory and practice and call for applications of portfolio theory that are accessible and useful to investors. Our study adopts this approach. Obtaining reliable, quantitative evidence on the 4% rule and alternative withdrawal rates is of critical importance given their widespread use.

Retirees must balance the desire to make larger withdrawals to maintain a reasonable standard of living against two risks that can deplete their wealth: longevity risk (i.e., the retiree's lifespan exceeds actuarial expectations) and return risk (i.e., poor real investment returns in retirement). Modeling the impact of a random US retiree's longevity on withdrawal rules is straightforward given actuarial information on mortality risk. Modeling return risk is a more difficult problem. Quantifying the likelihood and severity of left-tail outcomes is particularly important, as poor market performance during retirement can be catastrophic. In the withdrawal rule literature, returns are typically modeled using either the historical performance of US asset classes or simulations parameterized to match the historical moments of US returns.

There are, however, reasons to be cautious when using historical US data to generate ex ante expectations of long-horizon investment outcomes and left-tail risk. A large literature identifies three primary concerns. First, the sample size of long-horizon returns for major asset classes in the US is exceedingly small. The commonly used post-1925 data on equity returns from the Center for Research in Security Prices (CRSP), for example, represent fewer than four independent observations of 30-year returns for US stocks. Overlapping observations do little to increase the effective sample size [Boudoukh, Israel, and Richardson (2019)], so the US data provide limited statistical information about long-run asset performance. Second, the US data suffer from survivor bias [Brown, Goetzmann, and Ross (1995) and Jorion and Goetzmann (1999)]. That is, the design choice to formulate retirement advice using US data is influenced by the ex post success of the US financial markets, which is problematic for current and future retirees who are concerned with forward-looking performance. Third, the US sample is subject to an easy data bias [Dimson, Marsh, and Staunton (2002)] as researchers focus on markets that, ex post, have uninterrupted return data that are not obfuscated by trading halts, wars, hyperinflation, and other extreme events.

<sup>&</sup>lt;sup>6</sup>See Benartzi, Previtero, and Thaler (2011), MacDonald, Jones, Morrison, Brown, and Hardy (2013), and Gomes, Haliassos, and Ramadorai (2021) for reviews of the annuitization puzzle.

Consistent with the limitations of the US data, evidence suggests that (i) the historical US asset market performance exceeded ex ante expectations [see, e.g., Fama and French (2002); Avdis and Wachter (2017); and Binsbergen, Hua, and Wachter (2022)] and (ii) the experience of US investors does not mirror that of investors in many other developed markets [see, e.g., Jorion and Goetzmann (1999) and Anarkulova, Cederburg, and O'Doherty (2023)]. For example, although long-horizon equity market losses are rare or nonexistent in the US [e.g., Fama and French (2018)], Japan's stock market suffered a nominal (real) return of -9% (-21%) over the recent 30-year period from 1990 to 2019 (a period that began with Japan holding the top spot in the world in market capitalization). In sum, a substantial literature demonstrates that the historical record of asset-class performance in the US likely offers a poor reflection of the forward-looking return distribution.

In this study, we reevaluate the 4% rule for a US investor using a comprehensive new dataset of real returns for domestic equity, international equity, government bonds, and government bills in developed economies. The dataset is specifically constructed to combat the issues relating to small samples, survivor bias, and easy data bias that impact the conclusions of prior studies. The data cover approximately 2,500 years of asset-class returns in 38 developed countries over the period from 1890 to 2019. The long time series and broad cross-sectional coverage yield a wealth of statistical information about potential investment outcomes and allow for a more complete characterization of left-tail risk. Our fundamental premise is that US investors today can form better ex ante expectations using a broad developed market sample free of survivor and easy data biases rather than relying on the small, biased US sample.<sup>7</sup>

Using this dataset, we simulate retirement outcomes associated with alternative withdrawal rates. We model asset-class returns using a modification of the stationary block bootstrap of Politis and Romano (1994) that maintains both cross-sectional and time-series properties of long-horizon asset returns. We incorporate longevity risk into the simulation design using mortality tables from the Social Security Administration (SSA). Our base case simulation focuses on the joint investment-longevity outcomes for a couple retiring in 2022 at age 65 who chooses a portfolio strategy of 60% domestic stocks and 40% bonds.

The 4% rule proves woefully inadequate for current retirees. A retired couple faces a 17.4%

<sup>&</sup>lt;sup>7</sup>Pfau (2010) applies Bengen's (1994) approach of examining rolling, 30-year samples to estimate the maximum safe withdrawal rates in 17 countries from the Dimson, Marsh, and Staunton (2002) database. Pfau (2010) finds that the 4% rule would have led to financial ruin over several historical periods in countries outside the US. Pfau's (2010) research design does not model longevity risk and relies on perfect foresight asset allocation policies (for each country and 30-year period) that are not implementable in real time. As such, our study — designed to provide quantitative evidence on ex ante expectations and normative policy advice on retirement spending — addresses fundamentally different questions than Pfau (2010).

probability of depletion of financial wealth prior to death (henceforth, "financial ruin") using the 4% rule, such that there is nearly a one-in-five chance that they must subsist — often for many years — solely on social welfare programs. Given the poor performance of the 4% rule, we explore alternative constant real withdrawal policies. Our findings suggest that most retirees (i.e., retirees with relatively modest levels of wealth) cannot achieve a reasonable standard of living while maintaining a very low ruin probability. To achieve a 1% ruin probability, for example, retirees must adopt a withdrawal rate of just 0.80% (i.e., \$8,000 of withdrawals per year for \$1 million in savings). When we attempt to balance the desires to achieve a higher standard of living and to avoid financial ruin, we find that a retired couple willing to bear a 5% ruin probability may withdraw 2.26% per year. This value is considerably lower than those proposed in prior studies, and it is just over half of the 4.22% rate implied by the post-1925 US data.

We consider three strategy modifications to examine the robustness of these results. First, we evaluate the impact of allowing investors to deviate from the base case allocation of 60% stocks and 40% bonds by investing more or less aggressively in stocks. In most specifications, these alternative investment strategies imply lower withdrawal rates relative to the static 60/40 stock-bond portfolio. The estimated withdrawal rates based on the developed country sample are also materially lower than those based on the US sample across all the allocation strategies considered. Second, we examine whether the investment strategies pursued by popular target-date funds are useful in supporting higher withdrawal rates. Target-date investment vehicles use glide-path strategies that shift into more conservative asset classes as the investor ages with the stated purposes of preserving wealth and mitigating longevity risk. We find, however, that target-date funds do little or nothing to enhance the modest withdrawals implied by our base case design. Moreover, target-date strategies underperform relative to the 60/40 stock-bond portfolio in terms of generating wealth for bequests. Third, we consider two alternative retirement spending approaches: (i) a "guardrails" strategy that adjusts the payout rate as markets rise or fall and (ii) a constant payout rate of the current principal balance. Among potential spending rules, the 4% rule lies on one end of the spectrum given its rigid focus on maintaining constant real payouts at the risk of financial ruin, the constant percentage of wealth approach lies on the other end because it eliminates the risk of ruin by eschewing stability in real payouts, and the guardrails strategy lies between. Retirees who adopt any of these approaches fare significantly worse when judged using the developed market sample compared with using the US sample.

We also examine how future generations may fare as life expectancy increases. We use SSA mortality estimates for today's young adults (retirement in 2065) and newborns (retirement in

2085) and find that the increased longevity materially impacts the safe withdrawal rate. For a retired couple willing to accept a 5% chance of financial ruin, the real withdrawal rate of 2.26% for today's retirees drops to 2.02% for today's young adults and to 1.95% for today's newborns.

Although we define financial ruin as the retired couple depleting their savings, we recognize that most US retirees receive Social Security benefits (and a smaller, declining number receive income from defined benefit plans) that continue until death. But Social Security benefits for most retirees are modest.<sup>8</sup> The Social Security Administration advises retirees, "Social Security is not meant to be your only source of income in retirement. On average, Social Security will replace about 40% of your annual pre-retirement earnings..."<sup>9</sup> As such, retirees who deplete their savings or who must substantially reduce their withdrawals are likely to face a poor standard of living.

Finally, we focus on spending rates for the US, but our results on safe withdrawal rates generalize to other developed countries. In fact, longevity is higher in most developed countries compared with the US. According to the World Bank, the current life expectancy is 76.3 years in the US versus 80.4 years in the European Union, such that current EU retirees have life expectancy (and safe withdrawal rates) similar to US newborns.<sup>10</sup> Of course, the degree of hardship associated with depleting financial wealth will also differ across countries due to differences in national retirement benefits and social programs.

The remainder of the paper is organized as follows. Section 2 describes our approach to constructing a sample of asset-class returns for a broad cross section of developed countries. Section 3 details our bootstrap methods for estimating the joint distribution of household portfolio and longevity outcomes. Section 4 presents our empirical findings, and Section 5 concludes.

# 2 Data

The primary data for our study are a panel of monthly real returns for domestic stocks, international stocks, bonds, and bills for 38 developed countries compiled by Anarkulova, Cederburg, and O'Doherty (2023). The data cover the period from 1890 to 2019, but as detailed below, the start dates for individual countries differ based on development classification and data availability. The sample construction is designed to mitigate two biases that plague other studies of investment performance in developed markets. First, a survivor bias [Brown, Goetzmann, and Ross (1995)]

<sup>&</sup>lt;sup>8</sup>Approximately 84% of individuals over age 65 receive Social Security benefits [Dushi, Iams, and Trenkamp (2017)]. The average monthly Social Security benefit in 2022 for couples in which both receive benefits is \$2,734, and the average for widow(er)s is \$1,567 per month (https://www.ssa.gov/news/press/factsheets/colafacts2023.pdf).

<sup>&</sup>lt;sup>9</sup>See https://www.ssa.gov/myaccount/assets/materials/workers-61-69.pdf.

<sup>&</sup>lt;sup>10</sup>See https://data.worldbank.org/indicator/SP.DYN.LE00.IN for 2021 life expectancy estimates.

arises if one conditions on eventual economic outcomes in sample construction. Second, an easy data bias [Dimson, Marsh, and Staunton (2002)] follows from researchers' preference to use readily available data that are uninterrupted by exchange closures accompanying wars, financial crises, and other extreme events. The Anarkulova, Cederburg, and O'Doherty (2023) dataset is specifically constructed to mitigate these biases by using ex ante measures of economic development to select markets, infilling incomplete data from historical sources, and carefully treating exchange closure periods.

#### 2.1 Development classification

Developed countries are included in our sample following the development classification approach introduced in Anarkulova, Cederburg, and O'Doherty (2022). This approach mitigates survivor bias by relying on ex ante measures of economic development to determine the initial sample inclusion date for a given country. Prior to 1948, a country is added to the sample from the first year in which its labor share in agriculture drops below 50%. This classification method is motivated by evidence in the development economics literature that labor transitions from agriculture to manufacturing and services as an economy develops [see, e.g., Kuznets (1973)]. Starting in 1948, a country is added to the sample from the year in which it joins the Organisation for Economic Co-operation and Development (OECD) or its predecessor, the Organisation for European Economic Co-operation (OEEC).

Table I (columns 2-3) shows the development year and classification benchmark for each country. With just three exceptions, each country remains in the sample through 2019 following its initial development classification. The exceptions correspond to Argentina (reclassified as a developing economy in 1966), Chile (1970), and Czechoslovakia (1945). In all three cases, the decision to reclassify the country follows substantial modifications to political and economic regimes.<sup>11</sup> Chile, the Czech Republic, and Slovakia ultimately reenter the sample based on admission to the OECD.

#### 2.2 Asset-class returns

For each sample country, our panel of data contains monthly real returns on domestic stocks, international stocks, bonds, and bills over the developed period. The returns are measured in

<sup>&</sup>lt;sup>11</sup>Argentina is removed from the sample with the 1966 military coup that led to seven years of military dictatorship. Chile is removed in 1970 with the surprise election of Salvador Allende and his near immediate nationalization of most major industries. Czechoslovakia permanently closed its stock market with the nationalization of most companies following the World War II German occupation. In all cases, we carefully construct the returns earned by investors during these shocks. See Anarkulova, Cederburg, and O'Doherty (2022) for detailed descriptions of the events preceding reclassification for these countries.

the local currency, and the local country's inflation rate is used to adjust nominal returns to real returns. As such, the returns for a given country reflect the real investment outcomes of local investors.

The starting point for dataset construction is the GFD atabase from Global Financial Data. The GFD at abase contains long time series of total return indexes, price indexes, and dividend yields for stocks; yields for bonds and bills; consumer price indexes (CPIs); exchange rates; and total market capitalization for a broad set of countries. To mitigate easy data bias, Anarkulova, Cederburg, and O'Doherty (2022, 2023) undertake significant steps to clean and validate data, to fill in gaps in the GFD atabase using alternative sources [e.g., League of Nations reports, the St. Louis Federal Reserve, various central bank websites and statistical vearbooks for individual countries, and Jordà. Knoll, Kuvshinov, Schularick, and Taylor (2019), and to recreate the investor experience during major market disruptions. The authors also detail 35 historical instances in which stock exchanges close for extended periods and continuous monthly data are unavailable (e.g., the closure of the New York Stock Exchange in 1914 at the onset of World War I), and they construct monthly returns across these episodes to reflect economic outcomes for investors. The international stock returns for a given country are a market-capitalization-weighted investment in all non-domestic equity markets, and the return measurement incorporates exchange rate changes to capture currency risk. The bond returns are based on ten-year government bonds and reflect instances of sovereign default or bond exchange during the sample (e.g., the Greek default in 2012). The bill returns are primarily based on three-month government bills (with central bank rates, interbank rates, and other shortterm interest rates filling occasional gaps in the data). We refer readers to Anarkulova, Cederburg, and O'Doherty (2022, 2023) for details on return measurement, special data issues, and dataset validation.

Table I provides details on sample eligibility (columns 4-5) and sample coverage (columns 6-8) for each developed country period. All country-months that are included in the sample have valid returns for all four asset classes. The eligible sample period represents the ideal period over which we would have complete return data. For most countries, sample eligibility starts from the later of 1890 (i.e., the start of the paper's sample period) and the development classification year. We also require that a country has issued ten-year government bonds, and this requirement delays sample eligibility for nine countries.<sup>12</sup> The sample coverage results suggest that Anarkulova, Cederburg, and O'Doherty's (2023) approach to data construction achieves comprehensive representation of the full developed period. Of the 39 developed country periods (i.e., 38 developed countries with

<sup>&</sup>lt;sup>12</sup>Estonia has no ten-year bonds outstanding during its developed period and is therefore excluded from the dataset.

two developed periods for Chile), our dataset contains a full time series of monthly data for all four asset classes for 27 cases. In the remaining cases, any missing observations occur at the beginning of the country's sample, such that there are no unintentional data gaps in the middle or at the end of a given series. This feature is important for mitigating the impact of survivor and easy data biases. In total, we have complete data on domestic stocks, international stocks, bonds, and bills for 29,919 months of the 33,007 possible months, such that our dataset covers 91% of the eligible sample.

Table II reports the geometric mean and standard deviation of real returns for each sample country and for the pooled sample of all country-month observations. Although sample period differences cloud cross-country comparisons of asset-class returns, the US earns higher average domestic stock returns but lower average bond returns relative to most other countries. The US is not, however, an outlier based on average domestic stock or bond performance.

## 2.3 Mortality

Our modeling approach examines the joint distribution of longevity and portfolio outcomes for US retirees. We model mortality risk using the period life tables from the SSA.<sup>13</sup> Table III summarizes the distribution of remaining life expectancy in years for the last survivor from a 65year-old heterosexual couple at various retirement dates. Our base case scenario is investors retiring in 2022. The life expectancy is 24.7 years for a couple, but there is considerable uncertainty about longevity outcomes. The 5th percentile of time to death is 12.3 years, whereas the 95th percentile is 35.5 years. The table also shows distributional statistics for investors retiring in 2065 (young adults today) or 2085 (newborns today). The life expectancies are longer, such that longevity concerns are more acute for younger generations.

# 3 Methods

We use a Monte Carlo simulation approach to study portfolio outcomes as a function of the real withdrawal rate. Each draw from the simulation tracks the retirement account balance of a heterosexual couple from retirement at age 65 through death of the last survivor. We simulate portfolio outcomes using a stationary block bootstrap approach [Politis and Romano (1994)], in which the underlying data are the real returns on domestic stocks, international stocks, bonds, and bills for developed countries described in Section 2.2. We simulate longevity outcomes for the

<sup>&</sup>lt;sup>13</sup>See https://www.ssa.gov/oact/HistEst/PerLifeTables/2021/PerLifeTables2021.html.

couple using the SSA mortality data described in Section 2.3.

Two aspects of the bootstrap procedure are particularly important for maintaining realistic aspects of the data. First, we draw matched sets of returns on domestic stocks, international stocks, bonds, and bills from the same country-months to preserve the cross-sectional relations across assets. Second, we draw blocks of returns that span many consecutive months from a given country using the block bootstrap to preserve time-series relations in the data. Block lengths are drawn from a geometric distribution with an average block length of 120 months. These long blocks allow the simulation to reflect short-term characteristics such as persistent return volatility and long-term characteristics like mean reversion in stock returns.

We consider outcomes for withdrawal rules ranging from 0% to 6% per year. Our base case uses the 2022 SSA life tables and portfolio weights of 60% domestic stocks and 40% bonds (henceforth the 60/40 portfolio). The 60/40 portfolio is a common investment rule of thumb, is consistent with the well-known home bias in asset holdings [see, e.g., French and Poterba (1991)], and is a relatively successful investment allocation (as shown below). We also consider alternative simulation designs based on life expectancies for those retiring in 2065 or 2085; alternative stock-bond allocations ranging from 0% to 100% in stocks; allocations from a target-date fund that invests in domestic stocks, international stocks, bonds, and bills using a glide path that depends on the investor's age; and alternative payout policies that deviate from fixed real withdrawal rules.

For each set of parameters, we generate 1,000,000 simulation draws to estimate financial ruin probabilities and other measures of interest. Each iteration m = 1, ..., 1,000,000 has the following steps:

- 1. We draw the lifespans of a male, a female, and a couple (with lifespan equal to the greater of the male and female lifespans) using the SSA mortality tables. Mortality tables report death probabilities by age conditional on living until that age. Investors are assumed to retire when they turn 65. We use monthly conditional death probabilities and Bernoulli draws to determine whether death occurs in a given month. The monthly conditional death probabilities are calculated as one-twelfth of the annual probabilities. We simulate the monthly longevity process until both the male and female have died, and we denote the length in months of the longer of the two lifespans as  $T^{(m)}$ .
- 2. We draw monthly asset returns on domestic stocks, international stocks, bonds, and bills to form a time series of  $T^{(m)}$  months of returns using the stationary block bootstrap approach of Politis and Romano (1994).

- (a) We draw a random block size b from a geometric distribution with a probability parameter equal to the inverse of the desired average block length (120 months).
- (b) We randomly select a starting observation return vector for the block from the 29,919 country-month observations in the pooled sample. We denote this observation as

$$R_{i,t} = \begin{bmatrix} R_{i,t}^{Domestic \ stocks} & R_{i,t}^{International \ stocks} & R_{i,t}^{Bonds} & R_{i,t}^{Bills} \end{bmatrix},$$
(1)

where *i* indexes the country and *t* indexes the month. The return block draw is  $B_b = \{R_{i,t}, R_{i,t+1}, \ldots, R_{i,t+b-1}\}$  if country *i*'s sample contains return observations  $R_{i,t}$  through  $R_{i,t+b-1}$ . If not, then  $\{R_{i,t}, R_{i,t+1}, \ldots, R_{i,T}\}$ , where  $R_{i,T}$  is the last observation in country *i*'s sample, is insufficient to fill block  $B_b$ . In this case, we draw a random country *j* from the 39 developed periods. If country *j* has enough observations to fill the remainder of the block, the block is  $B_b = \{R_{i,t}, R_{i,t+1}, \ldots, R_{i,T}, R_{j,1}, R_{j,2}, \ldots, R_{j,b-(T-t+1)}\}$ . If not, country *j*'s observations are added to the block, and we repeat the process and draw another random country until the block is filled.

- (c) We add  $B_b$  to the bootstrap return matrix draw  $R^{(m)}$ . We return to step 2(a) and repeat the process until the return matrix has  $T^{(m)}$  months of data for the four assets. The final bootstrap draw in iteration m is  $R^{(m)} = \{R_1^{(m)}, R_2^{(m)}, \ldots, R_{T^{(m)}}^{(m)}\}$ .
- 3. We calculate the monthly retirement account balances through death of the last survivor of the couple. The retirement portfolio returns depend on the row vector of portfolio weights  $w_t$ , where t indexes the month of the investor's retirement period and  $w_t$  contains weights for domestic stocks, international stocks, bonds, and bills. We consider seven sets of portfolio weights. Six of these seven maintain constant portfolio allocations across domestic stocks and bonds. We evaluate portfolio allocations of 100% bonds, 20% domestic stocks and 80% bonds, 40% stocks and 60% bonds, 60% stocks and 40% bonds, 80% stocks and 20% bonds, and 100% stocks. The seventh weighting scheme follows the glide path of a representative target-date fund, such that these weights change through retirement. Denoting the row vector of four asset returns in month t + 1 of retirement as  $R_{t+1}^{(m)}$ , the portfolio return is  $R_{p,t+1}^{(m)} = R_{t+1}^{(m)}w_t'$ . Given initial wealth  $W_0$  and adopted withdrawal rule r, the couple withdraws  $I = \frac{W_0 r}{12}$  at the beginning of each month. Because all returns in our sample are real returns, this retirement income is expressed in real terms. Real retirement wealth evolves as

$$W_{t+1}^{(m)} = \max(W_t^{(m)} - I, 0) R_{p,t+1}^{(m)}.$$
(2)

Given the monthly retirement account balances, we calculate various quantities including a dummy variable for financial ruin (e.g.,  $W_{t+1}^{(m)} = 0$  for  $t + 1 \leq T^{(m)}$ ), dummy variables for living for a certain number of months after financial ruin, and retirement wealth at death of the last survivor.

We use the Monte Carlo simulation to estimate probabilities of financial ruin, probabilities of outlasting retirement wealth and living a certain number of months, and quantiles of ending wealth ratios (i.e., ratios of real wealth at the time of death of the last survivor to real wealth at retirement) by aggregating across the 1,000,000 draws for each set of parameter values.

# 4 Results

We examine joint investment-longevity results as a function of the real withdrawal rate. Our base case in Section 4.1 focuses on a retirement-age, US couple in 2022 who implements a portfolio allocation of 60% domestic stocks and 40% bonds. We consider alternative stock-bond weighting schemes in Section 4.2 and a target-date fund weighting scheme in Section 4.3. We examine retirement dates corresponding to younger investors in Section 4.4. We study alternative retirement withdrawal strategies in Section 4.5. For completeness, we report results for single retirees (i.e., 65-year-old females and males) in the appendix.

#### 4.1 Base case

We begin our analysis with the base case of a 65-year-old couple in 2022 who holds the 60/40 portfolio. Figure 1 plots the likelihood that the couple fully depletes their retirement savings prior to the death of the last survivor (i.e., the probability of financial ruin) as a function of the real withdrawal rate. The blue line corresponds to our base case simulation design using the full sample of developed country returns. As a benchmark, the red line corresponds to a simulation that relies solely on the US data for the period from 1926 to 2019. The start of this US data series matches the start of the widely used CRSP data. Similar US-centric samples have been used extensively to calibrate inputs for retirement simulation exercises, but suffer from survivor and easy data biases. A comparison of the full-sample and US results yields quantitative evidence on the impact of these biases on retirement planning.

The simulation results suggest that Bengen's (1994) original 4% rule exposes investors to considerable risk of outliving their wealth in retirement. The 4% rule leads to financial ruin in 17.4% of simulation runs. Bengen's subsequently updated 5% rule has a failure rate of 29.8%, and even

the more conservative 3.3% rule recently proposed by Benz, Ptak, and Rekenthaler (2021) leads to an 11.1% probability of ruin.<sup>14</sup> Each of these failure rates likely exceeds a comfortable level of risk for most households.

In contrast, a model based on the historical US experience leads to a more encouraging view of the popular withdrawal plans. The failure rate for the 4% rule is 3.5%, and the failure rate for the 3.3% rule is just 0.8%. The results in Figure 1 thus highlight the pitfalls of using small historical samples and ignoring survivor and easy data biases in evaluating retirement withdrawal policies.

Given the inadequacy of popular rules for facilitating a safe retirement, we explore alternative withdrawal rates. We find that there is no withdrawal rate that both supports a reasonable standard of living for most retirees and nearly assures they will not outlive their wealth. Allowing for a 1% ruin probability, for example, implies a withdrawal rate of just 0.80%, which provides only \$667 per month of income for each \$1 million in savings. With the modest retirement savings balances of most retirees, this low withdrawal rate provides little help in maintaining a reasonable standard of living.

Retirees are therefore faced with a tradeoff. Larger withdrawals enhance income and consumption, but smaller withdrawals alleviate the risk of financial ruin. To further characterize this tradeoff, we move away from "safe" withdrawal rates and consider retirees who are willing to bear a higher risk of financial ruin. For example, Figure 1 shows that the withdrawal rate corresponding to a 5% failure probability estimated from the developed sample is 2.26% for couples retiring in 2022. The corresponding rate estimated from the US sample is 4.22%. The economic implications of the disparity in these rates are significant. For example, the median income in 2020 for US households ages 55-64 was \$76,631 [Shrider, Kollar, Chen, and Semega (2021)]. To maintain this level of income (recognizing that income may differ from spending) in retirement, a 2.26% withdrawal rate implies that a couple receiving the average monthly Social Security benefit of \$2,734 needs \$1.94 million in retirement savings [i.e., ( $$76,631-12 \times $2,734$ )/0.0226] rather than the \$1.04 million implied by a 4.22% real payout.<sup>15</sup>

Our broad-based developed country sample also has implications for projected wealth accumulation. Figure 2 summarizes the distribution of the ending wealth ratio (i.e., real wealth at the time

<sup>&</sup>lt;sup>14</sup>See https://www.marketwatch.com/story/the-inventor-of-the-4-rule-just-changed-it-11603380557 for discussion of Bengen's 5% rule. Both the 5% rule and Benz, Ptak, and Rekenthaler's (2021) 3.3% rule condition on the current macroeconomic environment (i.e., asset valuation levels, interest rates, and inflation).

<sup>&</sup>lt;sup>15</sup>Prior research suggests that, in contrast to the consumption-smoothing behavior predicted by standard life-cycle models, household consumption declines in retirement. Been, Rohwedder, and Hurd (2021), for example, estimate a 16% reduction in spending. This behavior and other factors would impact the required level of savings at the onset of retirement.

of death of the last survivor from the couple divided by real wealth at retirement) based on the developed sample (blue) and the US sample (red) as a function of the real withdrawal rate. For each sample, the solid line corresponds to the median ending wealth ratio, and the shaded region covers the 10th to 90th percentiles of the ratio. Both samples suggest considerable uncertainty about real ending wealth. For a couple willing to bear a 5% probability of ruin (i.e., one with a 2.26% withdrawal rate), for example, the 10th percentile of the ending wealth ratio for the developed sample simulation is 0.25, whereas the 90th percentile is 5.43. The most pronounced feature of the plot, however, is the downward shift of the distribution based on the developed sample relative to the distribution based on the US sample. For instance, at a 4% withdrawal rate, the US data suggest that median ending wealth is 62% higher than starting retirement wealth. In contrast, the median ending wealth outcome is 9% lower than starting retirement wealth based on the developed sample.

The implications of the elevated financial ruin probabilities estimated from the developed sample in Figure 1 are particularly acute if retirees experience ruin well before death. Figure 3 shows the joint probability (estimated from the developed sample) that a couple fully depletes their retirement wealth and the last survivor lives at least X additional years  $[X \in \{1, 5, 10\}]$  as a function of the real withdrawal rate. The results reveal that the economic consequences of adopting an overly aggressive withdrawal rate are severe. With the 4% rule, for example, there is a 16.0% (11.1%) [6.1%] chance that retirees will experience financial ruin and live one (five) [ten] or more years after exhausting their retirement wealth.

### 4.2 Alternative stock-bond allocation strategies

Our base case design focuses on the withdrawal policies of couples retiring in 2022 with a 60/40 portfolio. Panel A of Table IV reports estimated withdrawal rates for ruin probabilities of 1%, 5%, and 10% based on the developed sample and the US sample for alternative stock-bond allocations. The table shows withdrawal rates for 2022 retiring couples with stock-bond allocations ranging from 0% stocks and 100% bonds to 100% stocks and 0% bonds; the results for 60% stocks and 40% bonds correspond to the base case from Section 4.1. The analysis yields no evidence that retirees can materially increase their withdrawal rates by shifting from the 60/40 portfolio to invest more heavily in stocks or bonds. Moreover, the differences in results for the developed sample and the US sample suggest that accounting for survivor and easy data biases in financial asset returns remains important across all allocation strategies and tolerances for likelihood of financial ruin.

## 4.3 Target-date funds

Target-date funds (also known as life-cycle funds) have grown from approximately \$900 billion in 2014 to \$3.3 trillion in 2021 [Pacholok and Zaya (2022)], and more than half of 401(k) participants hold at least one target-date fund [Holden, VanDerhei, and Bass (2021)]. A target-date fund implements a more aggressive portfolio when the investor is young to facilitate wealth accumulation. The strategy shifts toward a more conservative portfolio as the investor nears retirement age and typically becomes increasingly conservative through retirement years with the intended purposes of preserving wealth and mitigating risks from longevity and poor investment performance.

Figure 4 shows the glide-path weights for a representative target-date fund from a major investment advisor. During the investor's early working years, the fund allocates 90% of the portfolio to domestic and international stocks and 10% to bonds. The stock weights begin to decrease about 20 years prior to the retirement date and continue to fall through the early retirement period. The allocation 30 years after the retirement date is 17% to domestic and international stocks and 83% to bonds and bills.

Panel B of Table IV shows that the target-date strategy implies lower withdrawal rates compared with the 60/40 portfolio for failure probabilities of 1%, 5%, and 10%. For example, whereas our base case generates a 2.26% withdrawal rate for a 5% probability of ruin, the target-date strategy supports just 2.14%. This result generalizes. Figure 5 presents the probability of financial ruin as a function of the real withdrawal rate for the 60/40 and target-date fund strategies. The ruin probabilities are estimated using the developed sample for a couple retiring in 2022. The analysis offers no evidence that target-date funds support higher withdrawal rates by reducing longevity and return risks. The target-date strategy increases the likelihood of financial ruin relative to the 60/40 portfolio for each withdrawal rate we consider.

Figure 6 plots the distributions of the ending wealth ratio for the 60/40 strategy (blue) and the target-date strategy (red) based on the developed country sample. The solid lines denote median ending wealth levels, and the shaded regions cover the 10th through 90th percentiles of wealth outcomes. The target-date strategy also performs poorly relative to the 60/40 portfolio in terms of wealth accumulation through retirement. The median and 90th percentile of the ending wealth ratio for the 60/40 portfolio are uniformly above the corresponding values for the target-date allocation, and the 10th percentiles of the two distributions are nearly identical. In short, our evidence suggests that target-date funds do not support higher withdrawal rates, considerably limit upside wealth accumulation, and fail to enhance downside protection.

# 4.4 Alternative retirement dates

The SSA predicts that life expectancies will increase over time. For example, Table III shows that the median longevity for a couple retiring in 2085 is 15% longer than that for a couple retiring in 2022 (28.9 years versus 25.2 years). As a result, for any risk level the corresponding withdrawal rate falls over time. To quantify the effects, Panel C of Table IV reports results for investors with retirement dates of 2065 (young adults today) and 2085 (newborns today). The 2.26% withdrawal rate for a 5% ruin probability in 2022 declines to 2.02% for 2065 retirees and to 1.95% for 2085 retirees.

## 4.5 Alternative withdrawal strategies

Although the 4% rule is the most commonly advised approach to retirement spending [Choi (2022)], practitioners and researchers have proposed several alternative rules with variable real spending rates. Although these alternatives reduce the risk of financial ruin by design, they do so at the cost of an increased likelihood that the retiree faces (potentially severe) spending cuts following poor financial market returns.

In the appendix, we study two such alternatives: (i) the "guardrails" strategy of Guyton and Klinger (2006) and Klinger (2016), in which the real withdrawal amount declines if the portfolio does poorly and increases if the portfolio does well, and (ii) a constant withdrawal percentage of the value of the portfolio in each month. The 4% rule and the constant proportion of wealth strategy lie on opposite ends of the withdrawal rule spectrum given their different focuses on maintaining stability in real withdrawals versus eliminating the risk of financial ruin, and the guardrails approach lies between. As such, the performance of these policies provides an encompassing view of potential approaches to retirement spending.

Analogous to our evaluation of the 4% rule, expected outcomes under these alternative withdrawal strategies are much worse with the developed market sample versus the US sample. For example, a guardrail strategy using an initial 4% rate faces a 6.8% likelihood of financial ruin based on the full sample compared with just a 0.2% likelihood with the US sample. The developed sample also implies greater risk of severe spending cuts resulting from poor financial market returns. In a similar vein, even though the constant percentage of wealth approach ensures that retirees never exhaust their savings (i.e., a couple only experiences financial ruin if their portfolio return is -100%), they face much greater risks over retirement consumption and ending wealth based on the developed market data. A couple withdrawing an annualized 4% of the portfolio balance has a 49% chance of an average real withdrawal amount that is less than the 4% initial payout level, compared with only a 29% likelihood using US data.<sup>16</sup> In short, regardless of the strategy under consideration, investors forming expectations from the small, biased US sample will be overly optimistic relative to those forming expectations from the broad sample of developed markets.

# 5 Discussion and conclusions

We study withdrawal rules for retirees, bringing to bear a wealth of information on the historical performance of major asset classes in developed countries. Our comprehensive new dataset mitigates the survivor and easy data biases that plague prior work. We find that there is no withdrawal rate that allows most retirees to maintain a reasonable standard of living while being virtually assured they will not outlive their wealth. Even if a couple is willing to bear a 5% ruin probability, the withdrawal rate is just 2.26%. This modest withdrawal rate implies that households must accumulate substantial savings to avoid severe spending cuts during retirement.

Beyond its advice for individuals, our study has implications for society and public policy. Retirees in a given cohort experience the same asset returns in retirement, such that financial ruin outcomes driven by poor investment performance are highly correlated. Under the prevailing 4% rule, our estimates suggest that generations of retirees jointly face great risk of financial ruin. Financial ruin for a cohort (or multiple consecutive cohorts) of retirees from an extended period of poor returns may lead to widespread poverty among the elderly and increased enrollment in meanstested social programs (e.g., Medicaid and Section 8 housing). As such, overestimating the safe withdrawal rate effectuates both individual and societal costs. When combined with the growing threat of Social Security's insolvency [Board of Trustees (2021)] and the defined benefit pension's demise, popular retirement spending rules place the three-legged stool of retirement [e.g., Poterba (2014)] in a precarious position. Our findings emphasize the need to develop and popularize simple, yet robust, tools to strengthen the role of savings in retirement, lest we awaken to find that the three-legged stool hasn't a leg to stand on.

<sup>&</sup>lt;sup>16</sup>Yet another approach to retirement spending is a self-annuitization strategy via a Treasury Inflation Protected Securities (TIPS) ladder. There are several limitations to this approach, however, that may explain why the strategy remains relatively unpopular compared with the 4% rule. The payout rates provided by a TIPS ladder depend on real yields at retirement (which are unknown until that time), so savers face the risk that real yields will be low or negative (as was the case for much of the past two decades). TIPS held in taxable accounts also have unfavorable tax treatment, and taxable retirees may be forced to unwind the ladder in inflationary periods to meet intermediate tax obligations. Finally, a TIPS ladder ensures financial ruin and no bequest at its ending date, such that retirees are exposed to substantial longevity risk. Regardless, retirees have a revealed preference for spending rules relative to (self or commercial) annuitization strategies.

# References

- Anarkulova, Aizhan, Scott Cederburg, and Michael S. O'Doherty, 2022, Stocks for the long run? Evidence from a broad sample of developed markets, *Journal of Financial Economics* 143, 409–433.
- ——, 2023, Long-horizon losses in stocks, bonds, and bills, Working paper, Emory University.
- Avdis, Efstathios, and Jessica A. Wachter, 2017, Maximum likelihood estimation of the equity premium, Journal of Financial Economics 125, 589–609.
- Been, Jim, Susann Rohwedder, and Michael Hurd, 2021, Households' joint consumption spending and home production responses to retirement in the US, *Review of Economics of the Household* 19, 959–985.
- Benartzi, Shlomo, Alessandro Previtero, and Richard H. Thaler, 2011, Annuitization puzzles, Journal of Economic Perspectives 25, 143–164.
- Bengen, William P., 1994, Determining withdrawal rates using historical data, Journal of Financial Planning 7, 171–180.
- Benz, Christine, Jeffrey Ptak, and John Rekenthaler, 2021, The state of retirement income: Safe withdrawal rates, <a href="https://www.morningstar.com/lp/the-state-of-retirement-income">https://www.morningstar.com/lp/the-state-of-retirement-income</a>, Morningstar Research.
- Binsbergen, Jules H. van, Sophia Hua, and Jessica A. Wachter, 2022, Is the United States a lucky survivor: A hierarchical Bayesian approach, Working paper, University of Pennsylvania.
- Board of Trustees, 2021, The 2021 Annual Report of the Board of Trustees of the Federal Old-age and Survivors Insurance and Federal Disability Insurance Trust Funds, https://www.ssa.gov/ oact/TR/2021/tr2021.pdf.
- Boudoukh, Jacob, Ronen Israel, and Matthew Richardson, 2019, Long-horizon predictability: A cautionary tale, *Financial Analysts Journal* 75, 17–30.
- Brown, Jeffrey R., James Poterba, and David Richardson, 2023, Trends in retirement and retirement income choices by TIAA participants: 2000–2018, Forthcoming in *Journal of Pension Economics and Finance*.
- Brown, Stephen J., William N. Goetzmann, and Stephen A. Ross, 1995, Survival, Journal of Finance 50, 853–873.
- Choi, James J., 2022, Popular personal financial advice versus the professors, Journal of Economic Perspectives 36, 167–192.
- Cochrane, John H., 2022, Portfolios for long-term investors, Review of Finance 26, 1–42.
- Cooley, Philip, Carl Hubbard, and Daniel Walz, 1998, Retirement savings: Choosing a withdrawal rate that is sustainable, *AAII Journal* 20, 16–21.
- Cooley, Philip L., Carl M. Hubbard, and Daniel T. Walz, 2003, Does international diversification increase the sustainable withdrawal rates from retirement portfolios?, *Journal of Financial Planning* 16, 74–81.

- Davidoff, Thomas, Jeffrey R. Brown, and Peter A. Diamond, 2005, Annuities and individual welfare, American Economic Review 95, 1573–1590.
- Dimson, Elroy, Paul R. Marsh, and Michael Staunton, 2002, Triumph of the Optimists: 101 Years of Global Investment Returns. Princeton University Press, Princeton, NJ.
- Dushi, Irena, Howard M. Iams, and Brad Trenkamp, 2017, The importance of Social Security benefits to the income of the aged population, *Social Security Bulletin* 77, 1–12.
- Fama, Eugene F., and Kenneth R. French, 2002, The equity premium, *Journal of Finance* 57, 637–659.
- ——, 2018, Long-horizon returns, Review of Asset Pricing Studies 8, 232–252.
- French, Kenneth R., and James M. Poterba, 1991, Investor diversification and international equity markets, *American Economic Review: Papers & Proceedings* 81, 222–226.
- Gomes, Francisco, Michael Haliassos, and Tarun Ramadorai, 2021, Household finance, *Journal of Economic Literature* 59, 919–1000.
- Guyton, Jonathan T., and William J. Klinger, 2006, Decision rules and maximum initial withdrawal rates, *Journal of Financial Planning* 19, 49–57.
- Holden, Sarah, Jack VanDerhei, and Steven Bass, 2021, Target date funds: Evidence points to growing popularity and appropriate use by 401(k) plan participants, https://www.ebri.org/docs/default-source/ebri-issue-brief/ebri\_ib\_537\_ 401ktdfs-9sep21.pdf?sfvrsn=bf653b2f\_4, EBRI Issue Brief.
- Jordà, Oscar, Katharina Knoll, Dmitry Kuvshinov, Moritz Schularick, and Alan M. Taylor, 2019, The rate of return on everything, 1870–2015, *Quarterly Journal of Economics* 134, 1225–1298.
- Jorion, Philippe, and William N. Goetzmann, 1999, Global stock markets in the twentieth century, Journal of Finance 54, 953–980.
- Klinger, William J., 2016, Guardrails to prevent potential retirement portfolio failure, Journal of Financial Planning 29, 46–53.
- Kuznets, Simon, 1973, Modern economic growth: Findings and reflections, American Economic Review 63, 247–258.
- Lockwood, Lee M., 2018, Incidental bequests and the choice to self-insure late-life risks, American Economic Review 108, 2513–2550.
- MacDonald, Bonnie-Jeanne, Bruce Jones, Richard J. Morrison, Robert L. Brown, and Mary Hardy, 2013, Research and reality: A literature review on drawing down retirement financial savings, North American Actuarial Journal 17, 181–215.
- Mitchell, Olivia S., James M. Poterba, Mark J. Warshawsky, and Jeffrey R. Brown, 1999, New evidence on the money's worth of individual annuities, *American Economic Review* 89, 1299–1318.
- Pacholok, Megan, and Karen Zaya, 2022, 2022 target-date strategy landscape, https://www.morningstar.com/lp/tdf-landscape, Morningstar Research.

- Pfau, Wade D., 2010, An international perspective on safe withdrawal rates: The demise of the 4% rule?, *Journal of Financial Planning* 23, 52–61.
- Politis, Dimitris N., and Joseph P. Romano, 1994, The stationary bootstrap, Journal of the American Statistical Association 89, 1303–1313.
- Poterba, James, Steven Venti, and David Wise, 2011, The composition and drawdown of wealth in retirement, *Journal of Economic Perspectives* 25, 95–118.
- Poterba, James M., 2014, Retirement security in an aging population, *American Economic Review* 104, 1–30.
- Pye, Gordon B., 2000, Sustainable investment withdrawals, *Journal of Portfolio Management* 26, 73–83.
- Reichling, Felix, and Kent Smetters, 2015, Optimal annuitization with stochastic mortality and correlated medical costs, *American Economic Review* 105, 3273–3320.
- Scott, Jason S., William F. Sharpe, and John G. Watson, 2009, The 4% rule At what price?, Journal of Investment Management 7, 31–48.
- Shrider, Emily A., Melissa Kollar, Frances Chen, and Jessica Semega, 2021, Income and poverty in the United States: 2020, https://www.census.gov/content/dam/Census/library/ publications/2021/demo/p60-273.pdf, United States Census Bureau Report Number P60-273.
- Yaari, Menahem, 1965, Uncertain lifetime, life insurance, and the theory of the consumer, *Review of Economic Studies* 32, 137–150.

# TABLE I – DEVELOPED COUNTRY SAMPLE PERIODS

The table shows developed countries, initial development dates, classification reasons for development, sample eligibility details, and sample coverage. The development year classifications are based on agricultural labor share or membership in the Organisation for European Economic Co-operation (OEEC) or the Organisation for Economic Co-operation (OEEC) or the Organisation for Economic table to the country has issued long-term government bonds. The sample period start date is the later of the eligibility date and the first date with return data for stocks, bonds, and bills. Coverage is the percentage of the eligible sample period with complete return data for a given country.

	D	evelopment	San	ple eligibility	Sa	mple cove	rage
Country	Year	Classification	Year	Cause of delay	Start	End	Coverage
United Kingdom	1841	Labor share	1890	Sample start	1890:01	2019:12	100.0
Netherlands	1849	Labor share	1890	Sample start	1914:01	2019:12	81.5
Belgium	1856	Labor share	1890	Sample start	1897:01	2019:12	94.6
France	1866	Labor share	1890	Sample start	1890:01	2019:12	100.0
Norway	1875	Labor share	1890	Sample start	1914:02	2019:12	81.5
Germany	1882	Labor share	1890	Sample start	1890:01	2019:12	100.0
Denmark	1890	Labor share	1890	_	1890:01	2019:12	100.0
Switzerland	1890	Labor share	1890	—	1914:01	2019:12	81.5
United States	1890	Labor share	1890	—	1890:01	2019:12	100.0
Canada	1891	Labor share	1891	—	1891:01	2019:12	100.0
Argentina	1895	Labor share	1895	—	1947:02	1966:12	27.7
New Zealand	1896	Labor share	1896	_	1896:01	2019:12	100.0
Australia	1901	Labor share	1901	_	1901:01	2019:12	100.0
Sweden	1910	Labor share	1910	_	1910:01	2019:12	100.0
Austria	1920	Labor share	1920		1925:02	2019:12	94.9
Chile period I	1920	Labor share	1920	_	1927:01	1970:12	86.3
Greece	1920	Labor share	1920	_	1981:02	2019:12	38.9
Czechoslovakia	1921	Labor share	1921		1926:01	1945:05	79.5
Japan	1930	Labor share	1930	_	1930:01	2019:12	100.0
Portugal	1930	Labor share	1930	_	1934:01	2019:12	95.6
Italy	1931	Labor share	1931	_	1931:01	2019:12	100.0
Ireland	1936	Labor share	1936	_	1936:01	2019:12	100.0
Singapore	1947	Labor share	1998	Bonds	1998:07	2019:12	100.0
Iceland	1948	OEEC	1992	Bonds	2002:01	2019:12	64.3
Luxembourg	1948	OEEC	1948	—	1982:01	2019:12	52.8
Türkiye	1948	OEEC	2010	Bonds	2010:02	2019:12	100.0
Spain	1959	OEEC	1959	—	1959:01	2019:12	100.0
Finland	1969	OECD	1969		1969:01	2019:12	100.0
Mexico	1994	OECD	2001	Bonds	2001:08	2019:12	100.0
Czech Republic	1995	OECD	2000	Bonds	2000:05	2019:12	100.0
Hungary	1996	OECD	1999	Bonds	1999:02	2019:12	100.0
Poland	1996	OECD	1999	Bonds	1999:06	2019:12	100.0
South Korea	1996	OECD	2000	Bonds	2000:11	2019:12	100.0
Slovakia	2000	OECD	2000	_	2000:01	2019:12	100.0
Chile period II	2010	OECD	2010	_	2010:01	2019:12	100.0
Estonia	2010	OECD		Bonds			
Israel	2010	OECD	2010	_	2010:01	2019:12	100.0
Slovenia	2010	OECD	2010		2010:01	2019:12	100.0
Latvia	2016	OECD	2016	_	2016:01	2019:12	100.0
Lithuania	2018	OECD	2018		2018:01	2019:12	100.0

## TABLE II – SUMMARY STATISTICS

The table reports summary statistics for monthly real returns for domestic stocks, international stocks, bonds, and bills for each developed country and for the pooled sample of all developed countries. The table shows the number of sample months (Obs) for each country and the geometric mean return (Mean) and the standard deviation of return (StDev) for each country and asset class.

			nestic ocks		ational cks	Bo	nds	Bi	Bills		
Country	Obs	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev		
Argentina	239	-0.18	8.53	0.64	15.34	-1.66	2.84	-1.56	2.73		
Australia	1,428	0.58	3.90	0.42	3.76	0.16	1.68	0.07	0.54		
Austria	$1,\!139$	0.27	5.18	0.57	12.43	0.16	2.67	-0.00	1.50		
Belgium	1,476	0.22	5.01	0.38	4.54	0.04	1.76	-0.03	1.14		
Canada	1,548	0.48	4.24	0.42	3.47	0.19	1.62	0.12	0.57		
Chile period I	528	0.13	6.15	0.62	8.49	-0.92	3.38	-0.86	2.34		
Chile period II	120	-0.03	4.06	0.78	3.54	0.14	1.37	0.03	0.36		
Czech Republic	236	0.86	7.07	-0.00	4.18	0.25	2.16	-0.04	0.43		
Czechoslovakia	233	-0.45	6.89	0.25	6.23	0.30	3.03	0.10	2.87		
Denmark	1,560	0.33	3.54	0.38	3.90	0.23	1.85	0.18	0.72		
Finland	612	0.78	6.31	0.41	4.31	0.32	2.21	0.06	0.46		
France	1,560	0.30	5.40	0.42	6.67	-0.06	2.27	-0.16	1.77		
Germany	1,560	0.26	8.35	0.56	10.26	-0.12	46.30	0.17	0.86		
Greece	467	0.45	10.36	0.54	4.71	0.36	5.55	0.16	1.27		
Hungary	251	0.46	6.44	0.26	4.08	0.40	3.31	0.18	0.40		
Iceland	216	-0.07	7.66	0.31	4.86	0.36	3.30	0.23	0.53		
Ireland	1,008	0.46	4.67	0.47	4.03	0.20	2.38	0.03	0.59		
Israel	120	-0.06	4.81	0.66	3.36	0.59	1.86	0.11	0.85		
Italy	1,068	0.17	7.41	0.44	13.15	-0.12	2.54	-0.25	1.71		
Japan	1,080	0.30	6.67	0.49	16.21	-0.18	3.47	-0.33	2.67		
Latvia	48	0.97	3.54	0.61	2.96	0.05	1.33	-0.21	0.47		
Lithuania	24	0.18	2.61	0.61	3.52	0.16	1.31	-0.21	0.49		
Luxembourg	456	0.58	5.50	0.58	4.47	0.39	1.76	0.13	0.58		
Mexico	221	0.67	4.75	0.53	3.53	0.39	2.55	0.15	0.38		
Netherlands	1,272	0.40	5.09	0.41	4.37	0.16	1.66	0.02	0.78		
New Zealand	1,488	0.50	3.65	0.43	4.09	0.15	1.80	0.15	0.59		
Norway	1,271	0.39	5.06	0.44	4.21	0.15	1.70	0.02	0.86		
Poland	247	0.32	5.98	0.23	3.66	0.44	2.48	0.22	0.41		
Portugal	1,032	0.13	7.92	0.45	4.03	0.05	2.80	-0.06	1.36		
Singapore	258	0.53	5.94	0.27	3.99	0.22	1.99	-0.02	0.47		
Slovakia	240	0.37	5.33	-0.06	4.13	0.49	2.89	-0.04	0.58		
Slovenia	120	0.29	4.03	0.86	3.18	0.45	2.98	-0.02	0.76		
South Korea	230	0.70	6.15	0.27	3.73	0.37	1.90	0.09	0.34		
Spain	732	0.34	5.48	0.39	4.22	0.20	2.17	0.02	0.69		
Sweden	1,320	0.47	4.82	0.44	4.14	0.17	1.81	0.09	0.97		
Switzerland	1,272	0.39	4.31	0.38	4.46	0.16	1.38	0.03	0.62		
Türkiye	119	0.05	6.44	1.13	5.04	0.00	4.88	0.06	0.94		
United Kingdom	1,560	0.38	4.28	0.45	4.09	0.16	1.93	0.07	0.87		
United States	1,560	0.52	4.99	0.33	3.78	0.14	1.73	0.06	0.61		
<full sample=""></full>	29,919	0.37	5.59	0.43	6.74	0.10	10.81	0.00	1.17		

TABLE III – DISTRIBUTION OF TIME TO DEATH

The table summarizes the distribution of life expectancy in years of the last survivor from a 65-year-old heterosexual couple at various retirement dates. The distributions are constructed using the actuarial life tables from the Social Security Administration.

Moments			Percentiles								
Retirement	Mean	StDev	1%	5%	10%	25%	50%	75%	90%	95%	99%
2022	24.7	7.0	6.8	12.3	15.3	20.3	25.2	29.5	33.3	35.5	39.9
2065	27.6	7.2	8.9	15.1	18.4	23.3	28.0	32.4	36.4	39.0	43.7
2085	28.7	7.2	9.8	16.3	19.5	24.2	28.9	33.4	37.7	40.3	45.0

#### TABLE IV - WITHDRAWAL RATES

The table shows the real payout rate in percentage estimated from 1,000,000 bootstrap simulations for various asset allocation strategies, retirement dates, and underlying data samples (i.e., the full sample of developed countries and the post-1925 US sample). The base case design (denoted with a  $\dagger$ ) corresponds to an asset allocation strategy of 60% stocks and 40% long-term bonds and a retirement date of 2022. The alternative cases modify either the asset allocation strategy or the retirement date, as described in the table. For each design, the table reports withdrawal rates corresponding to 1%, 5%, and 10% probabilities of financial ruin. The investor type is a heterosexual couple, such that financial ruin is defined as exhausting financial resources prior to death of the last survivor.

			Pr(Financi	al Ruin)				
	1%		5%		10%	10%		
Description	Developed sample	US sample	Developed sample	US sample	Developed sample	US sample		
Panel A:	Withdrawal r	ates for co	nstant stock-bo	ond allocati	ons			
0% stocks / 100% bonds 20% stocks / 80% bonds 40% stocks / 60% bonds 60% stocks / 40% bonds (†) 80% stocks / 20% bonds 100% stocks / 0% bonds	0.14 0.33 0.58 0.80 0.87 0.66 3: Withdrawa	1.77 2.90 3.27 3.39 3.25 2.87	1.02 1.64 2.06 2.26 2.21 1.85 target-date fun	2.84 3.53 4.01 4.22 4.20 3.97	$2.20 \\ 2.69 \\ 3.02 \\ 3.15 \\ 3.11 \\ 2.82$	$3.23 \\ 3.93 \\ 4.48 \\ 4.77 \\ 4.79 \\ 4.62$		
Target-date fund weights	0.72	3.05	2.14	3.72	3.12	4.15		
Panel C	: Withdrawa	l rates for a	alternative retin	rement date	es			
2065 retirement date 2085 retirement date	$\begin{array}{c} 0.71 \\ 0.68 \end{array}$	$3.19 \\ 3.13$	$2.02 \\ 1.95$	$4.00 \\ 3.93$	$2.86 \\ 2.77$	$\begin{array}{c} 4.51 \\ 4.43 \end{array}$		

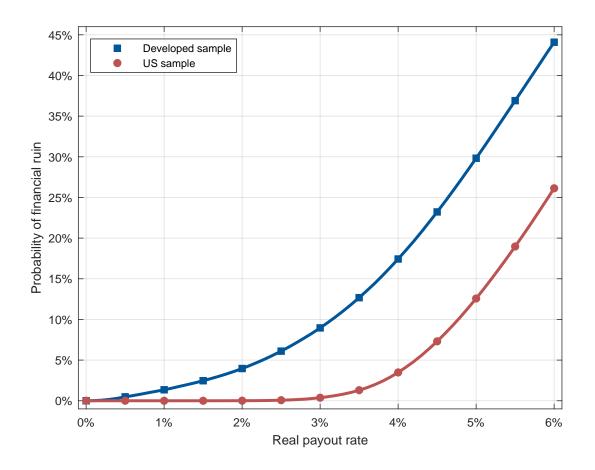


FIGURE 1 – PROBABILITY OF FINANCIAL RUIN. The figure plots the likelihood that a 65-year-old couple fully depletes their retirement wealth prior to the death of the last survivor as a function of the real withdrawal rate. The assumed asset allocation mix is 60% stocks and 40% bonds, and the survival probabilities are based on the 2022 actuarial life tables from the Social Security Administration. The blue (red) line shows the proportion of bootstrap simulations using the pooled sample of all developed countries (post-1925 US sample) in which wealth reaches zero prior to the death of the last survivor. We simulate 1,000,000 joint portfolio return and household survival outcomes for each combination of sample and withdrawal rate.

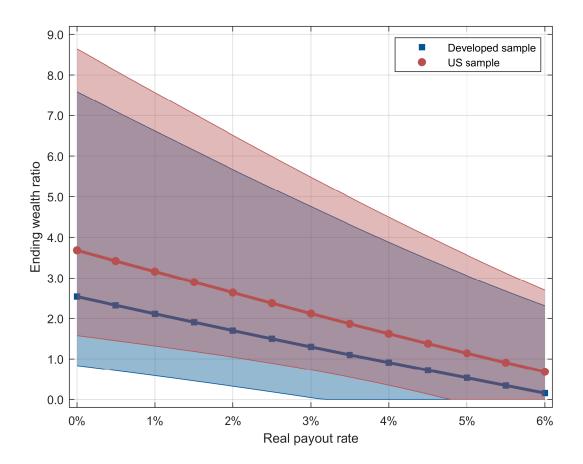


FIGURE 2 – DISTRIBUTION OF ENDING WEALTH RATIO. The figure plots the distribution of the ratio of real ending wealth to real wealth at the start of retirement for a 65-year-old couple as a function of the real withdrawal rate. Real ending wealth is the real wealth at the time of death of the last survivor. The assumed asset allocation mix is 60% stocks and 40% bonds, and the survival probabilities are based on the 2022 actuarial life tables from the Social Security Administration. The blue (red) plot corresponds to the distribution of the ending wealth ratio across bootstrap simulations using the pooled sample of all developed countries (post-1925 US sample). The solid lines indicate the median ending wealth ratios, and the shaded regions covers the 10th to 90th percentiles of the distributions. We simulate 1,000,000 joint portfolio return and household survival outcomes for each combination of sample and withdrawal rate.

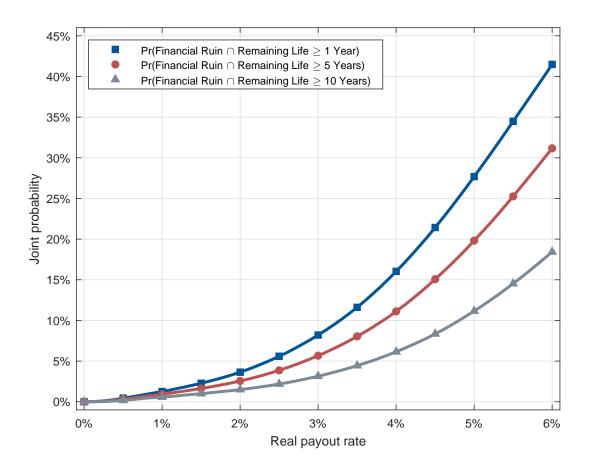


FIGURE 3 – REMAINING LIFE UPON FINANCIAL RUIN. The figure plots the joint likelihood that a 65-year-old couple fully depletes their retirement wealth prior to the death of the last survivor and the last survivor lives at least X additional years as a function of the real withdrawal rate. The assumed asset allocation mix is 60% stocks and 40% bonds. The survival probabilities are based on the 2022 actuarial life tables from the Social Security Administration, and the underlying sample for the bootstrap simulations is the pooled sample of all developed countries. The blue (red) [gray] line shows the joint probability of financial ruin and one member of the couple living at least one (five) [ten] years after ruin. We simulate 1,000,000 joint portfolio return and household survival outcomes for each withdrawal rate.

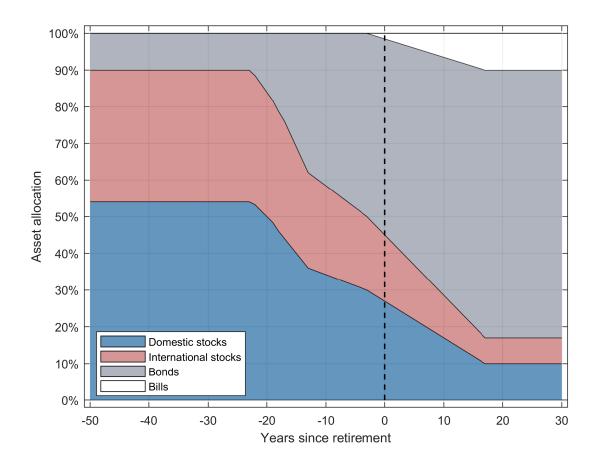


FIGURE 4 – TARGET-DATE FUND GLIDE-PATH WEIGHTS. The figure shows the approximate asset allocation of a representative target-date fund across domestic stocks, international stocks, bonds, and bills as a function of time since retirement.

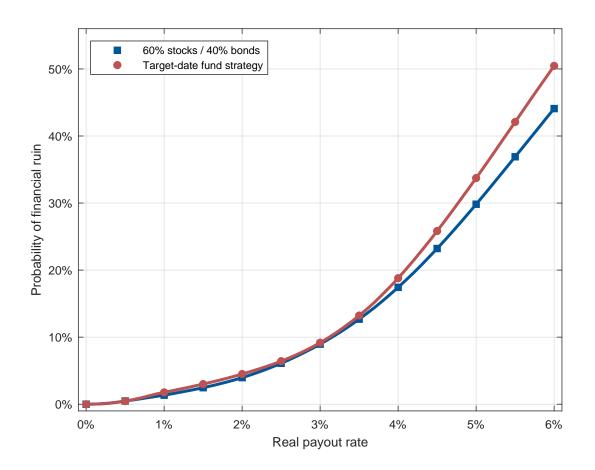


FIGURE 5 – PROBABILITY OF FINANCIAL RUIN: COMPARISON OF 60/40 STRATEGY WITH TARGET-DATE FUND STRATEGY. The figure plots the likelihood that a 65-year-old couple fully depletes their retirement wealth prior to the death of the last survivor as a function of the real withdrawal rate for alternative asset allocation strategies. The blue (red) line shows the probability of financial ruin for the investment strategy of 60% stocks and 40% bonds (the investment strategy implied by the glide path for a representative target-date fund). The survival probabilities are based on the 2022 actuarial life tables from the Social Security Administration, and the underlying sample for the bootstrap simulations is the pooled sample of all developed countries. We simulate 1,000,000 joint portfolio return and household survival outcomes for each combination of asset allocation strategy and withdrawal rate.

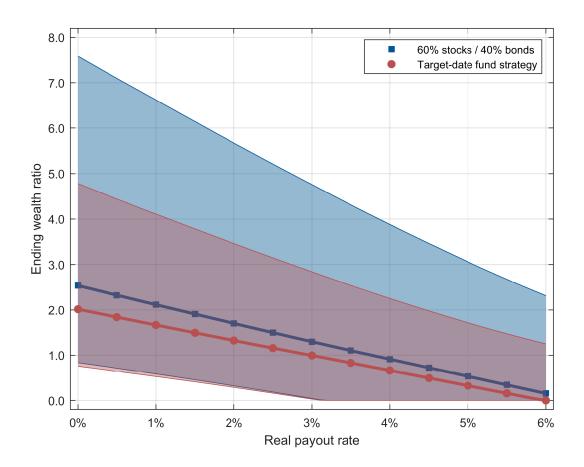


FIGURE 6 – DISTRIBUTION OF ENDING WEALTH RATIO: COMPARISON OF 60/40 STRATEGY WITH TARGET-DATE FUND STRATEGY. The figure plots the distribution of the ratio of real ending wealth to real wealth at the start of retirement for a 65-year-old couple as a function of the real withdrawal rate for alternative asset allocation strategies. Real ending wealth is the real wealth at the time of death of the last survivor. The blue (red) plot corresponds to the investment strategy of 60% stocks and 40% bonds (the investment strategy implied by the glide path for a representative target-date fund). The survival probabilities are based on the 2022 actuarial life tables from the Social Security Administration, and the underlying sample for the bootstrap simulations is the pooled sample of all developed countries. For each distribution, the solid line indicates the median ending wealth ratio and the shaded region covers the 10th to 90th percentiles of the distribution. We simulate 1,000,000 joint portfolio return and household survival outcomes for each combination of asset allocation strategy and withdrawal rate.

# A Supplementary appendix

In this appendix, we present additional empirical results.

## A.1 Single retirees

Our main results in Section 4 correspond to joint investment-longevity outcomes for 65-year-old couples as a function of the real portfolio withdrawal rate, the retirement date, and the portfolio investment strategy. This appendix provides the corresponding results for single females and single males.

Table A.I presents distributional statistics for remaining life expectancy for 65-year-old females and 65-year-old males at various retirement dates. Table A.II shows safe real withdrawal rates for these investor types for various combinations of asset allocation strategy, tolerance for likelihood of financial ruin, and underlying data sample.

Figure A.1 shows the probability of financial ruin as a function of the real withdrawal rate for single females and males, and Figure A.2 summarizes the distribution of ending wealth for these investor types.

## A.2 Alternative withdrawal policies

We focus in Section 4 on rules that specify a constant real withdrawal amount because of the ubiquity of the 4% rule. This appendix considers two alternative withdrawal strategy designs:

1. Guardrails approach: Given initial wealth  $W_0$  and adopted withdrawal rule r, the couple withdraws  $I_0 = \frac{W_0 r}{12}$  at the beginning of each month in the first year. In subsequent years, the real monthly withdrawal amount  $I_t$  potentially increases or decreases depending on portfolio performance following the strategy outlined by Klinger (2016). Monthly real retirement wealth evolves as

$$W_{t+1} = \max(W_t - I_t, 0)R_{p,t+1}.$$
(A1)

At the beginning of each year from the second year of retirement through the 15th year, with the first month of the given year indexed as t, the couple evaluates whether the previous withdrawal amount  $I_{t-1}$  remains appropriate given their current wealth level  $W_t$ . If  $I_{t-1} >$  $1.2 \times \frac{W_t r}{12}$  (i.e., the status quo withdrawal amount is more than 20% higher than a withdrawal amount calculated from current wealth and the adopted withdrawal rule), then  $I_t = 0.9 \times I_{t-1}$ (i.e., the withdrawal rate is reduced by 10%). Symmetrically, the withdrawal amount increases by 10% if current wealth is sufficiently high, i.e., if  $I_{t-1} < 0.8 \times \frac{W_t r}{12}$  then  $I_t = 1.1 \times I_{t-1}$ . The couple then maintains constant real withdrawal amounts until death. An exception to this evolution of withdrawal amounts occurs in cases with very low financial wealth or financial ruin (i.e.,  $W_t < I_t$  or  $W_t = 0$ ), as the actual payout is equal to  $\max(I_t, W_t)$  in each month.

2. Constant proportion of wealth approach: Given initial wealth  $W_0$  and adopted withdrawal rule r, the couple withdraws  $I_0 = \frac{W_0 r}{12}$  in the first month. In all subsequent months, the couple withdraws  $I_t = \frac{W_t r}{12}$ . Monthly real retirement wealth evolves as

$$W_{t+1} = \max(W_t - I_t, 0)R_{p,t+1}.$$
(A2)

Financial ruin only occurs if the gross portfolio return  $R_{p,t+1}$  equals zero, i.e., the net portfolio return is -100% in a given month. Given our sample of data and portfolio strategies that do not take levered or short positions, the gross portfolio return is always positive such that financial ruin occurs with zero probability.

Figures A.3 to A.5 show results for the guardrails strategy with initial withdrawal rates ranging from 0% to 6%. Each figure summarizes retirement outcomes using the developed country sample (blue) and the US sample (red). Figure A.3 plots the probability of financial ruin as a function of the initial real withdrawal rate, and Figure A.4 summarizes the distribution of ending wealth. Given that the guardrails strategy makes adjustments to monthly real withdrawal amounts based on portfolio performance, the average real withdrawal rate as a percentage of initial wealth at retirement could be lower or higher than the initial withdrawal rate. Figure A.5 summarizes the distribution of the average real withdrawal rate.

Figures A.6 and A.7 present results for the constant proportion of wealth strategy. The financial ruin probability is 0% for this strategy, so we summarize the distributions of ending wealth (Figure A.6) and the average real withdrawal rate as a percentage of initial wealth at retirement (Figure A.7).

TABLE A.I – DISTRIBUTION OF TIME TO DEATH FOR SINGLE INVESTORS The table summarizes the distribution of life expectancy in years of a 65-year-old female (Panel A) and a 65-year-old male (Panel B) at various retirement dates. The distributions are constructed using the actuarial life tables from the Social Security Administration.

Moments				Percentiles							
Retirement	Mean	StDev	1%	5%	10%	25%	50%	75%	90%	95%	99%
Panel A: Females retiring at age 65											
2022	20.8	8.9	1.1	4.6	7.9	14.6	21.6	27.3	31.8	34.3	39.2
2065	23.4	9.3	1.6	6.2	10.2	17.5	24.3	30.0	34.8	37.5	42.8
2085	24.4	9.4	1.8	6.8	11.1	18.5	25.2	30.9	35.8	38.7	43.9
Panel B: Males retiring at age 65											
2022	18.2	8.8	0.7	3.0	5.6	11.7	18.8	24.8	29.5	32.0	36.7
2065	21.2	9.3	1.0	4.3	7.7	14.8	22.1	27.9	32.6	35.3	40.6
2085	22.2	9.4	1.1	4.8	8.6	15.9	23.1	28.9	33.7	36.6	42.1

#### TABLE A.II – WITHDRAWAL RATES FOR SINGLE INVESTORS

The table shows the real payout rate in percentage estimated from 1,000,000 bootstrap simulations for various asset allocation strategies, retirement dates, and underlying data samples (i.e., the full sample of developed countries and the post-1925 US sample). The base case design (denoted with a †) corresponds to an asset allocation strategy of 60% stocks and 40% long-term bonds and a retirement date of 2022. The alternative cases modify either the asset allocation strategy or the retirement date, as described in the table. For each design, the table reports withdrawal rates corresponding to 1%, 5%, and 10% probabilities of financial ruin. The investor type in Panel A (Panel B) is a 65-year-old female (65-year-old male).

			Pr(Financia	al Ruin)		
	1%		5%		10%	0
Description	Developed sample	US sample	Developed sample	US sample	Developed sample	US sample
	Panel A:	Females re	tiring at age 6	5		
A.1. W	Vithdrawal rat	es for cons	tant stock-bond	l allocations		
0% stocks / $100%$ bonds	0.18	2.38	1.40	3.04	2.62	3.49
20% stocks / $80%$ bonds	0.41	3.03	2.02	3.73	3.10	4.20
40% stocks / $60%$ bonds	0.70	3.42	2.42	4.23	3.42	4.78
60% stocks / $40%$ bonds (†)	0.96	3.55	2.59	4.47	3.55	5.10
80% stocks / $20%$ bonds	1.02	3.43	2.52	4.46	3.51	5.14
100% stocks / $0%$ bonds	0.75	3.06	2.15	4.23	3.24	4.98
A.2.	Withdrawal	rates for ta	rget-date fund	allocation		
Target-date fund weights	0.84	3.18	2.54	3.94	3.51	4.43
A.3.	Withdrawal r	ates for alt	ernative retirer	nent dates		
2065 retirement date	0.83	3.35	2.32	4.23	3.23	4.82
2085 retirement date	0.79	3.29	2.23	4.16	3.13	4.73
	Panel B	: Males reti	iring at age 65			
B.1. W	Vithdrawal rat	es for cons	tant stock-bond	l allocations		
0% stocks / $100%$ bonds	0.24	2.57	1.76	3.29	3.00	3.81
20% stocks / $80%$ bonds	0.48	3.22	2.37	3.98	3.47	4.52
40% stocks / $60%$ bonds	0.82	3.62	2.76	4.49	3.80	5.10
60% stocks / $40%$ bonds (†)	1.11	3.74	2.90	4.73	3.93	5.43
80% stocks / $20%$ bonds	1.17	3.63	2.82	4.71	3.87	5.47
100% stocks / $0%$ bonds	0.84	3.24	2.45	4.48	3.61	5.31
<i>B.2</i> .	Withdrawal n	rates for ta	rget-date fund	allocation		
Target-date fund weights	0.99	3.39	2.91	4.20	3.90	4.76
B.3.	Withdrawal r	ates for alt	ernative retirer	nent dates		
2065 retirement date	0.93	3.50	2.53	4.42	3.50	5.04
2085 retirement date	0.88	3.42	2.42	4.32	3.36	4.92

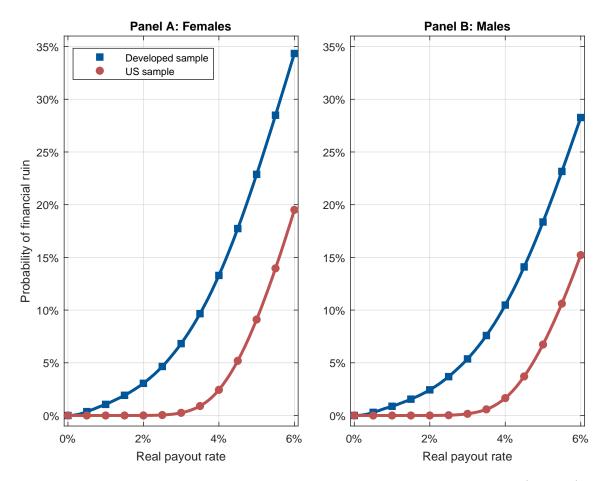


FIGURE A.1 – PROBABILITY OF FINANCIAL RUIN FOR SINGLE INVESTORS. Panel A (Panel B) plots the likelihood that a 65-year-old female (65-year-old male) fully depletes retirement wealth prior to the death as a function of the real withdrawal rate. The assumed asset allocation mix is 60% stocks and 40% bonds, and the survival probabilities are based on the 2022 actuarial life tables from the Social Security Administration. The blue (red) line shows the proportion of bootstrap simulations using the pooled sample of all developed countries (post-1925 US sample) in which wealth reaches zero prior to death. We simulate 1,000,000 joint portfolio return and survival outcomes for each combination of sample and withdrawal rate.

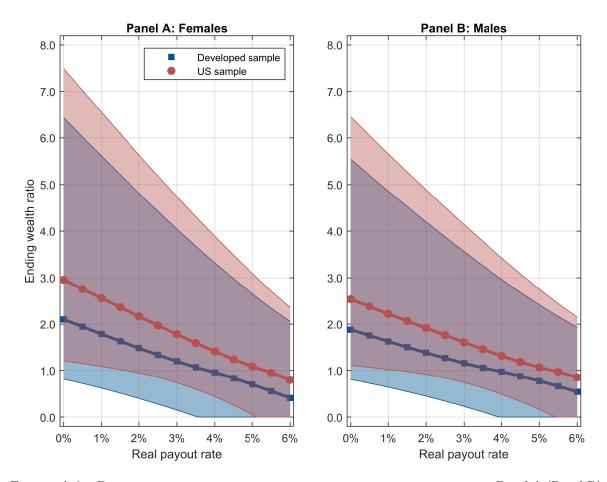


FIGURE A.2 – DISTRIBUTION OF ENDING WEALTH RATIO FOR SINGLE INVESTORS. Panel A (Panel B) plots the distribution of the ratio of real ending wealth to real wealth at the start of retirement for a 65-year-old female (65-year-old male) as a function of the real withdrawal rate. Real ending wealth is the real wealth at the time of death. The assumed asset allocation mix is 60% stocks and 40% bonds, and the survival probabilities are based on the 2022 actuarial life tables from the Social Security Administration. The blue (red) plot corresponds to the distribution of the ending wealth ratio across bootstrap simulations using the pooled sample of all developed countries (post-1925 US sample). The solid lines indicate the median ending wealth ratios, and the shaded regions covers the 10th to 90th percentiles of the distributions. We simulate 1,000,000 joint portfolio return and household survival outcomes for each combination of sample and withdrawal rate.

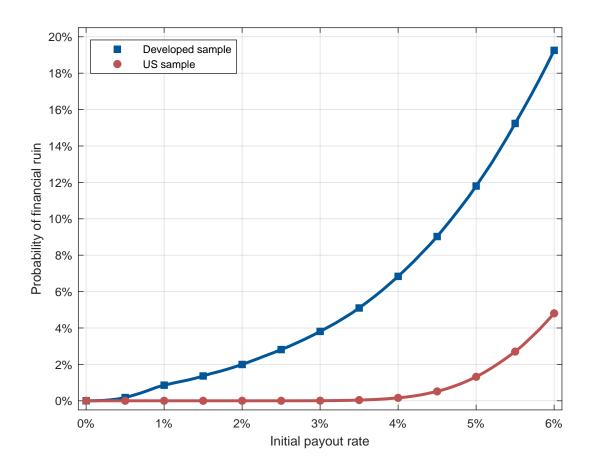


FIGURE A.3 – PROBABILITY OF FINANCIAL RUIN FOR GUARDRAILS STRATEGY. The figure plots the likelihood that a 65-year-old couple fully depletes their retirement wealth prior to the death of the last survivor as a function of the initial withdrawal rate under the guardrails strategy. The assumed asset allocation mix is 60% stocks and 40% bonds, and the survival probabilities are based on the 2022 actuarial life tables from the Social Security Administration. The blue (red) line shows the proportion of bootstrap simulations using the pooled sample of all developed countries (post-1925 US sample) in which wealth reaches zero prior to the death of the last survivor. We simulate 1,000,000 joint portfolio return and household survival outcomes for each combination of sample and withdrawal rate.

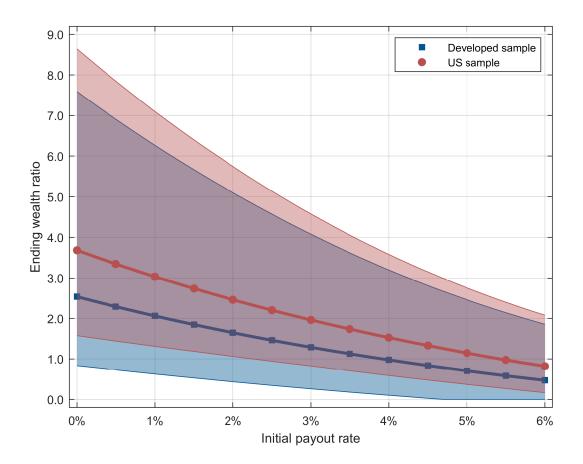


FIGURE A.4 – DISTRIBUTION OF ENDING WEALTH RATIO FOR GUARDRAILS STRATEGY. The figure plots the distribution of the ratio of real ending wealth to real wealth at the start of retirement for a 65-year-old couple as a function of the initial withdrawal rate under the guardrails strategy. Real ending wealth is the real wealth at the time of death of the last survivor. The assumed asset allocation mix is 60% stocks and 40% bonds, and the survival probabilities are based on the 2022 actuarial life tables from the Social Security Administration. The blue (red) plot corresponds to the distribution of the ending wealth ratio across bootstrap simulations using the pooled sample of all developed countries (post-1925 US sample). The solid lines indicate the median ending wealth ratios, and the shaded regions covers the 10th to 90th percentiles of the distributions. We simulate 1,000,000 joint portfolio return and household survival outcomes for each combination of sample and withdrawal rate.

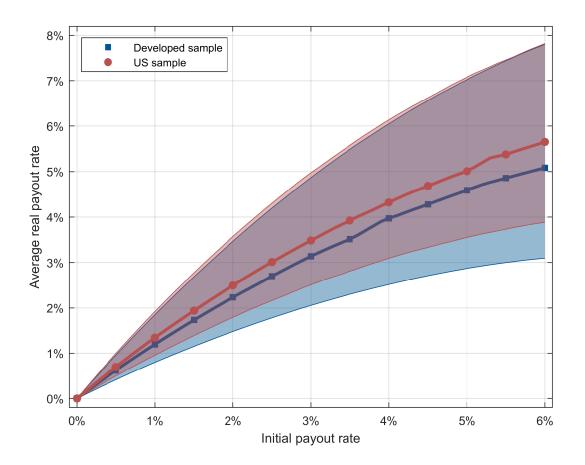


FIGURE A.5 – DISTRIBUTION OF AVERAGE REAL WITHDRAWAL RATE FOR GUARDRAILS STRATEGY. The figure plots the distribution of the average real withdrawal rate (as a proportion of wealth at the start of retirement) during retirement for a 65-year-old couple as a function of the initial withdrawal rate under the guardrails strategy. The assumed asset allocation mix is 60% stocks and 40% bonds, and the survival probabilities are based on the 2022 actuarial life tables from the Social Security Administration. The blue (red) plot corresponds to the distribution of the average real withdrawal rate across bootstrap simulations using the pooled sample of all developed countries (post-1925 US sample). The solid lines indicate the median average real withdrawal rates, and the shaded regions covers the 10th to 90th percentiles of the distributions. We simulate 1,000,000 joint portfolio return and household survival outcomes for each combination of sample and withdrawal rate.

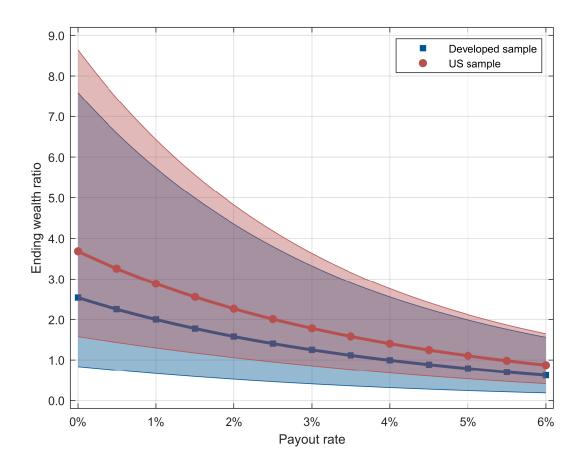


FIGURE A.6 – DISTRIBUTION OF ENDING WEALTH RATIO FOR CONSTANT PROPORTION OF WEALTH STRAT-EGY. The figure plots the distribution of the ratio of real ending wealth to real wealth at the start of retirement for a 65-year-old couple as a function of the withdrawal rate under the constant proportion of wealth strategy. Real ending wealth is the real wealth at the time of death of the last survivor. The assumed asset allocation mix is 60% stocks and 40% bonds, and the survival probabilities are based on the 2022 actuarial life tables from the Social Security Administration. The blue (red) plot corresponds to the distribution of the ending wealth ratio across bootstrap simulations using the pooled sample of all developed countries (post-1925 US sample). The solid lines indicate the median ending wealth ratios, and the shaded regions covers the 10th to 90th percentiles of the distributions. We simulate 1,000,000 joint portfolio return and household survival outcomes for each combination of sample and withdrawal rate.

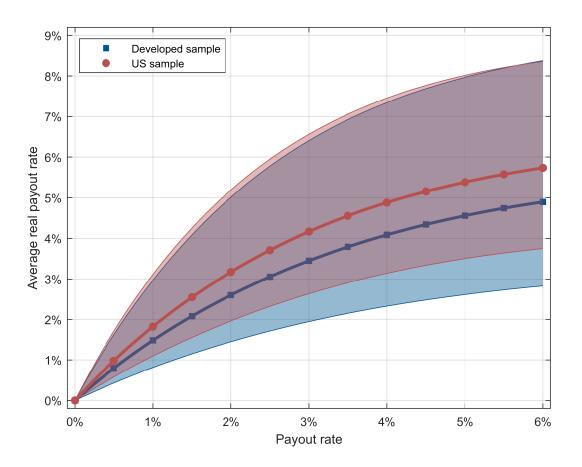


FIGURE A.7 – DISTRIBUTION OF AVERAGE REAL WITHDRAWAL RATE FOR CONSTANT PROPORTION OF WEALTH STRATEGY. The figure plots the distribution of the average real withdrawal rate (as a proportion of wealth at the start of retirement) during retirement for a 65-year-old couple as a function of the withdrawal rate under the constant proportion of wealth strategy. The assumed asset allocation mix is 60% stocks and 40% bonds, and the survival probabilities are based on the 2022 actuarial life tables from the Social Security Administration. The blue (red) plot corresponds to the distribution of the average real withdrawal rate across bootstrap simulations using the pooled sample of all developed countries (post-1925 US sample). The solid lines indicate the median average real withdrawal rates, and the shaded regions covers the 10th to 90th percentiles of the distributions. We simulate 1,000,000 joint portfolio return and household survival outcomes for each combination of sample and withdrawal rate.