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ENDLESS AND INFINITE

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It is often said that time must have a beginning because otherwise the series of past events would have the paradoxical features of an actual infinite. In the present paper, we show that, even given a dynamic theory of time, the cardinality of an endless series of events, each of which will occur, is the same as that of a beginningless series of events, each of which has occurred. Both are denumerably infinite. So if (as we believe) an endless series of events is possible, then the possibility of a beginningless series of past events should not be rejected merely on the ground that it would be an actual infinite. What would be required to rebut our argument is a symmetry breaker – something that motivates treating the past relevantly differently to the future. We consider several attempts to provide a symmetry breaker and show that none of them is successful.

Keywords: actual infinite, potential infinite, temporal becoming, presentism, Kalam cosmological argument.

I. INTRODUCTION

Proponents of the Kalam cosmological argument seek to establish that any temporally ordered series of discrete events must have a beginning. One of their principal arguments for this conclusion is that a beginningless series of discrete events would have the paradoxical features of an actual infinite – features that could not be instantiated 'in the real world'. In particular, they point out that an actually infinite series has a distinctive property, which we shall call the 'Cantorian Property'. A series has the Cantorian Property when it can be placed in one-to-one correspondence with infinitely many of its proper parts, so that the whole has the 'same number' of elements as its parts. For instance, there are just as many natural numbers as there are even numbers, etc. But in the 'real world', they say, the whole must always be greater than any of its proper parts. So, in the real world (as opposed to the world of mathematics), an actually infinite series is impossible; nothing real can have the Cantorian

Property (See Craig & Sinclair 2009: 110). And this is said to establish the first premise of the following argument:

1. An actual infinite cannot exist.
2. An infinite temporal regress of events is an actual infinite.
3. Therefore, an infinite temporal regress of events cannot exist. (Craig & Sinclair 2009: 103).

Now one might have thought that if these considerations were sufficient to show that a beginningless (and therefore infinite) series of past events is impossible, they would apply with equal force to an endless (and therefore infinite) series of future events.[1] After all, one could make a seemingly symmetrical argument as follows:

1. An actual infinite cannot exist.
2. An infinite temporal progress[2] of events is an actual infinite.
3. Therefore, an infinite temporal progress of events cannot exist.

If this second argument were equally as sound as the original one, this would be bad news for the proponents of the Kalam. For one thing, it is implausible to claim that the future could not be endless. For example, one can easily imagine a series of future events, each of which is causally sufficient for another. Again, one can imagine an endless series of events, each of which is fore-ordained by an all-powerful God. As far as we can see, these are genuine metaphysical possibilities.

Perhaps more importantly, many contemporary proponents of the Kalam argument believe in the life everlasting, where this is conceived as a never-ending sequence of heavenly or hellish experiences, each of which will occur. So if an endless series is infinite in the same sense as a beginningless one, these philosophers are in a bit of a bind. Either one of their key arguments for the impossibility of an infinite past is unsound, or an important tenet of their faith is not only false but metaphysically impossible. To get out of this bind, they need to come up with a 'symmetry breaker' – a reason for thinking that 'an infinite temporal *re*gress of events is an actual infinite' is *true*, but that 'an infinite temporal *pro*gress of events is an actual infinite' is *false*.

The most commonly heard proposal is that an endless series of future events differs from a beginningless series of past events in that it is a merely *potential infinite*, having none of the absurd implications of the actual infinite. It is this alleged difference that particularly interests us first. Is it the case that

[1] Authors who have suggested something along these lines include: Oppy 2006, Morriston 2002 and 2010, Hedrick 2014, and Cohen 2019.

[2] The term 'progress' is somewhat artificial, but is intended merely to be the future-orientated dual of 'regress'.

a beginningless series is an actual infinite, whereas an endless series would be only potentially infinite?

The answer might seem to depend on one's view of time. On an 'eternalist' view, there is no relevant difference between past and future events. Each occurs 'tenselessly' at its position in a temporally ordered series. 'Past' events are earlier, and 'future' events later, than whichever time is taken as a reference point. If for every discrete event there is an earlier one, then the series of 'past' events is an actual infinite; likewise, if for every discrete event there is a later one, then the series of 'future' events is an actual infinite.

However, proponents of the Kalam argument are typically not eternalists. They hold that tense is irreducible and temporal becoming real – so that there is an ever-changing fact of the matter about which events are present, and an equally objective fact about the direction of temporal becoming. On a view of this kind, there is one obvious difference between a beginningless series of past events and an endless series of future ones: the former has been completed, whereas the latter will never have been completed. From this, many friends of the Kalam argument conclude that the future is only potentially infinite. Thus William Lane Craig:

> . . . when we say that the number of future events is infinite, we do not mean that $\aleph_0$ events will elapse, for that is false. Ironically, then, it turns out that the series of future events cannot be actually infinite regardless of the infinity of the past or the metaphysical possibility of an actual infinite, for it is the objectivity of temporal becoming that *makes the future potentially infinite only* (Craig & Sinclair 2009: 116, emphasis added).

We don't agree. In Section II, we show that Craig's appeal to the potential infinite involves a fatal equivocation between *what will be* and *what will have been*. Once this equivocation has been exposed, it will be clear that, *even given a dynamic theory of time*, an endless series of distinct events is a countable infinite.

Sometimes, however, Craig offers a different symmetry breaker. Appealing to a presentist version of the A-theory, he claims that the number of future events cannot be infinite because future events don't exist or, alternatively, because they are mere 'potentialities'. In Sections III and IV, we show that neither of these theses about future events gives the slightest reason to think that an endless series of them is not denumerably infinite. In Section V, we dispose of a terminological worry about the expression 'actual infinite', and in Section VI, we turn to worries about inverse arithmetical operations on transfinite numbers 'in the real world'. We show that even if (contrary to what we believe) this consideration warranted us in rejecting the possibility of a beginningless past, it would do the same for an endless future.

Finally, in Section VII, we turn to an entirely different attempt to formulate a symmetry breaker – one that takes as a premise the claim that there cannot be an actual infinity of *presently* existing things (such as a Hilbert's Hotel). We'll assume (for the sake of argument) that this is so, and go on to consider

whether, as Andrew Loke has argued (Loke 2014), it has the implication that a beginningless series of past events is impossible. If it did, then we'd have an argument against the possibility of a beginningless past that could not be paralleled by an equally plausible argument against the possibility of an endless future. For those who think that a Hilbert's Hotel is impossible, this is quite an attractive line of argument, but we show that it contains a subtle but fatal flaw.

II. POTENTIAL VS ACTUAL INFINITY

We can see Craig's claim that the endless future is merely potentially infinite in his 2010 paper 'Taking Tense Seriously', when in response to a thought experiment from Morriston (2010) he writes:

> So with respect to Morriston's illustration of two angels who begin to praise God forever, an A-theorist will concur whole-heartedly with his statement, 'If you ask, 'How many distinct praises will be said?' the only sensible answer is, infinitely many'—that is to say, potentially infinitely many. If this answer is allowed the A-theorist, then Morriston's allegedly parallel arguments collapse. (Craig 2010: 452–453)

Thus, Craig is saying that the endless future is potentially infinite, and not actually infinite, which means that premise 2 of the second argument is false. If so, then Craig would have his 'symmetry breaker'.

However, we argue that Craig is not right about this. On the dynamic theory of time, *something* is potentially infinite, but that something is not the endless future. In order to explain this point, we need to spell out the context. Firstly, we need to get clear about what Craig means when he says that there will be 'potentially infinitely many' future events in an endless future. Then we will show how this notion plainly does not address the question of how many future events there will be. When we focus on the question in hand, it is clear that the answer is actually infinite.

But let's get clear on what the terms mean. If we look at the way Craig characterises the term 'potentially infinite', we can see that it involves the notions of increasing, having no limit, and always being finite.[3] For instance, in his book on the Kalam, Craig says the following about a potential infinite:

> Such a collection would be one in which the members are not definite in number but *may be increased without limit* (Craig 1979: 68–69, emphasis added).

[3] Craig's usage of the term 'potential infinite' is nowhere formally defined, and we have had to piece it together given various remarks he makes. The term 'potential infinite' doesn't have a canonical mathematical definition (Hart 1976), in contrast to the Cantorian understanding of the actual infinite. For an alternative way of characterising the notion which is formally rigorous, see Linnebo & Shapiro (2019).

A natural way to cash out this notion of increasing is in terms of *functions*; the potential future isn't a *set* as such, because sets have fixed membership. Craig clearly wants something more *dynamic*. In order to capture this dynamic idea of 'increasing over time', we use a function whose output increases as the input increases. And this seems to be how Craig sees it too, because in his paper 'Taking Tense Seriously', he describes the potentiality of the future also in terms of the value of a function approaching a limit; he says that if a series of events is potentially infinite then it is 'composed of a finite but ever-increasing number of events *with infinity as a limit*' (Craig 2010: 452, emphasis added).

Here is the most natural way to understand Craig's idea. Take the natural numbers in their usual ordering: (0, 1, 2, 3 . . .), and let them stand for distinct successive intervals of time. Let $A(x)$ be a function that takes numbers as its input (as values of the variable x) and returns the following class of numbers as its output: $\{y \mid y \leq x\}$. The output is *everything less than or equal to the input*. Thus, $A(2) = \{0, 1, 2\}$, and $A(5) = \{0, 1, 2, 3, 4, 5\}$, etc.

Two simple things follow immediately about this function. Firstly, if we increase x, then the cardinality of $A(x)$ similarly increases. The cardinality of $\{0, 1, 2, 3, 4, 5\}$ is greater than that of $\{0, 1, 2\}$, etc.

Secondly, let's call the output of $A(x)$ 'actually infinite' iff its elements can be placed in a one-to-one correspondence with the elements of one of its proper subsets (i.e. if it has the Cantorian property); call its output 'finite' iff it is not actually infinite (i.e. if it does not have the Cantorian property). It follows easily that:

Fact 1) For all values of x, the cardinality of $A(x)$ is *finite*

Whatever natural number we put into $A(x)$, the resulting class we get out is always going to have merely finitely many members. Thus, this fits with Craig's comments that 'the [potentially infinite] number of praises said by the angels will *always be finite*'.

The value of x can be any arbitrarily high number. There is no highest value that it can take. And this means that the cardinality of the class of numbers returned by $A(x)$ for different values of x also has no highest value. Thus, it also fits with Craig's comment that the members of a potential infinite 'may be increased without limit'.

As we saw, Craig sometimes describes the potential infinite as 'without limit', and sometimes 'with infinity as a limit'. These are not incompatible statements, as we take the former to implicitly mean 'without *finite* limit'. For those who prefer this way of talking (as Craig seems to – see, for example, Craig 2010: 455, n4.), we can express the way that the output of $A(x)$ has infinity as a limit as follows:

$$\lim_{x \to \infty} |A(x)| = \infty$$

All this says is that as the value of x approaches infinity, so does the cardinality of $A(x)$. This seems to capture Craig's notion of it increasing 'with infinity as a limit'. Given that the $A(x)$ function fits so well with the comments that Craig says about the potential infinite (increasing, without limit, and always finite), it seems reasonable to assume that this is the concept he has in mind.

But now, consider a different function. Let $B(x)$ be a function which takes natural numbers as its input, and has the following class as its output: $\{y \mid x < y\}$. The output is *everything greater than the input*. Thus, $B(2) = \{3, 4, 5 \ldots\}$, and $B(5) = \{6, 7, 8 \ldots\}$, etc.

A few simple things follow immediately about this function. First, given that there is no greatest natural number, as the value of x increases, the cardinality of $B(x)$ does *not*. The cardinality of $\{3, 4, 5 \ldots\}$ is *the same* as that of $\{6, 7, 8 \ldots\}$.

It also follows easily that:

Fact 2) For every input value of $B(x)$, its output is *actually infinite*

Whatever natural number we put into $B(x)$, the resulting class we get out is always going to have actually infinitely many members; that is, a set which can be put in a one-to-one correspondence with the natural numbers.

When Craig attempts to answer the question 'How many distinct praises will be said?' (or, equivalently: 'how many future events will there be if the future is endless?'), his reply is: 'potentially infinitely many'. Our contention is that he faces a dilemma: either what he says answers the right question but is false, or it is true but answers a different question. Either way, it is not satisfactory.

On the first horn of the dilemma, the answer provided by Craig seems obviously false. The number of events that will be is *not* 'a finite but ever-increasing number'. For example, it is not as if today there will be, say, ten future presidential inauguration events, but that tomorrow there will be eleven. However many future presidents will be sworn in, that number can only go down, as each new one gets sworn in. It certainly cannot *increase* as time passes. Given this truism, Craig's reply is just false.

However, we know what Craig means to express here. The trouble is that when you spell it out, we go down the second horn of the dilemma, and the answer doesn't address the original question. On the dynamic theory of time, it is not the future that is an 'ever-increasing number of events'; but rather it is *the past* that is ever-increasing. Take for instance the Aristotelian idea that potentialities become actualised as time passes; an idea that has much in common with the 'growing block' theory of time. According to this idea, as time passes, more and more events get added to the block of actual events, which 'grows'. Events that are currently future (and potential) will have been added to the stock of events that *have occurred*. And it is *this*, it seems to us, that

the output of the $A(x)$ function is giving us. If x is what time it currently is, then $A(x)$ is how many events *have occurred*.

But now the answer is not addressing the question. As we saw above, the question Craig took himself to be asking was: How many distinct praises *will be* said? The question is asking for the cardinality of those events that *will be*. But when Craig answers 'potentially infinitely many', he is saying that 'as time passes the number of events that *will have been* actualised increases without limit'. So we have changed tense, from simple future tense to the future perfect; Craig's answer is about what *will have been*, rather than what *will be*. This change of tense represents a change of subject; his answer would be relevant only if the question were 'how many events will have been?', but when we clarify that the question is 'how many events will there be?', his answer is plainly not relevant.

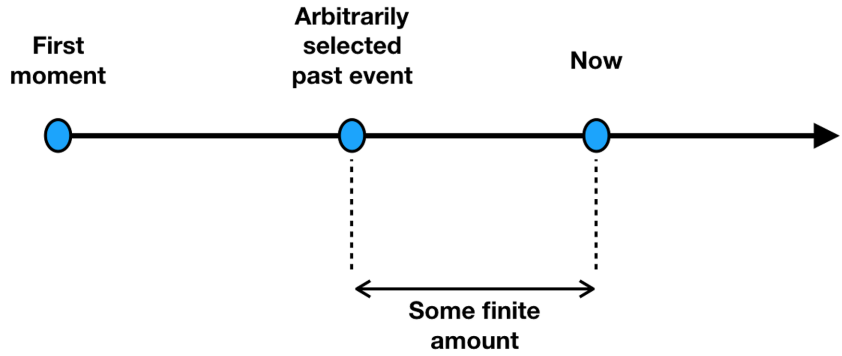
Craig seems to be grappling with this problem in the following passage.

> . . . one might be tempted to say that in an endless future there *will be* an actually infinite number of events, just as in an actually infinite past there *have been* an actually infinite number of events. But in a sense, that assertion is false; for there never will be an actually infinite number of events, since it is impossible to count to infinity. The only sense in which there will be an infinite number of events is that the series of events will go toward infinity as a limit. But that is the concept of a potential infinite, not an actual infinite. Here the objectivity of temporal becoming makes itself felt. For as a result of the arrow of time, *the series of events later than any arbitrarily selected past event* is properly to be regarded as a limit. The situation, significantly, is not symmetrical: as we have seen, the series of events earlier than any arbitrarily selected future event cannot properly be regarded as potentially infinite (Craig & Sinclair 2009: 116, emphasis ours).

Again, taken literally, it is not at all clear why 'the series of events later than any arbitrarily selected past event is properly to be regarded as a limit'; why should we think of it in terms of the $A(x)$ function at all? After all, 'the series of events later than any arbitrarily selected past event' does not increase in number as time passes at all (no matter how many inaugurations there will be later than some past event, that number cannot go up as time passes, etc).

Rather than thinking of it in terms of the $A(x)$ function, it is much more natural to think of it in terms of the $B(x)$ function. After all, 'the series of events *later than* any arbitrarily selected past event' is structurally analogous to the series of numbers *greater than* a given number, which is what the $B(x)$ function gives as its output. And, as we saw with fact 2, if the future is endless, the cardinality of the output of $B(x)$ is always actually infinite, not potentially infinite.

One way of reading Craig here that avoids this problem is to read him as saying that the number of events *between* some fixed past event and the moving now is finite but ever increasing towards infinity as a limit. If that's right, then he wants us to picture the situation this way:



His claim is that the *interval* between the past event and the present is always finite but increases with the passage of time. This is easily expressible as a two-place function on the natural numbers, $C(x, y)$, which returns the interval between x and y. If we keep x fixed and increase y, then the value of $C(x, y)$ has all the qualities that made the $A(x)$ function potentially infinite (it increases as y does, it has no finite limit, it is always finite, etc). So we have to read Craig as referring to the ever-increasing *interval between* some fixed point in the past and the location of the moving now for it to take on the properties of the potential infinite.

So, even though Craig said 'the series of events later than any arbitrarily selected past event is properly to be regarded as a limit', we have to take him to mean 'the series of events *in the interval between* an arbitrarily selected past event *and the present* is properly to be regarded as a limit'. It is right that this new phrase picks out an interval that is potentially infinite, but the problem is that it doesn't address the question of how many future events there *will be*. It addresses the gap between a past event and the present, which is just a complicated way of expressing the future perfect tense again; it is about how many future events *will have been*. Thus the reconstruction only succeeds in identifying something potentially infinite at the cost of not describing the future any longer.

Thus, Craig's answers are equivocal here. We ask a simple question about the cardinality of the extension of the $B(x)$ function: how many future events will there be? His answer can be interpreted in two ways. His answer is false if taken to be in terms of the $B(x)$ function. He says:

> . . . the series of events will go toward infinity as a limit. But that is the concept of a potential infinite, not an actual infinite.

But the extension of the $B(x)$ function is not 'increasing' at all, even if there is a moving now. It is not a limit and is not a potential infinite.

If, on the other hand, we understand him to be speaking in terms of the $A(x)$ function, then his reply is true but irrelevant to the question; the value of

the gap may be increasing – that is, the number of events there *will have been* after some past time may be increasing – but that simply isn't an answer to the question of 'how many events *will there be*?' It is just to answer a different question.

Thus, the symmetry breaker seems like it might succeed only because it changes the subject. If we stick to the issue at hand, the answer is obvious: if time has no end, there is an actually infinite number of events, each of which will be.

III. WHAT IF FUTURE EVENTS DON'T EXIST? CRAIG'S APPEAL TO PRESENTISM

Sometimes, however, Craig does give what appears to be an unambiguous answer to our question about the number of events, each of which will occur. He says that there are zero such events. According to his version of the A-theory, events aren't 'there' in the future, waiting to become present; they come into existence in the present, and then cease to exist. It follows that 'the only temporal entities that exist are present entities' (Craig 2001: 148). He sums up the implication of this view for the number of future events as follows.

> . . . on presentism there are no future events and so no series of future events. Therefore, the number of future events is simply zero, not 5_0 (Craig & Sinclair 2009: 116).

By this, Craig does not of course mean that there will be no more events. Indeed, the definition of presentism is expanded on by Craig as follows:

> Thus, there really are no past or future events, *except in the sense that there have been certain events and there will be certain others* (Craig 2001: 148, emphasis added).

But this qualification is all we need to make *our* case about the actual infinity of an endless future. Recall our insistence that the point to be addressed concerns the $B(x)$ function, which is the set of future events. Even if future events don't exist (because of presentism), they *will do* (because this is not the end of time). Thus, it seems to us that even if one grants Craig's premise, that only the present exists, this does not stop it being true that *each* of an actual infinity of events *will* exist.

So, does Craig perhaps think that there is something wrong with referring to (and numbering) events that don't exist? Not at all. He thinks there is no need for the complicated semantical manoeuvres made by other presentists. As long as we don't take our quantifiers or singular terms as 'devices of ontological commitment', he says that we can refer to, quantify over, and make true statements about objects and events that don't exist (Craig 2016: 118). Consequently, one can sensibly refer to past events, and their non-existence is 'no hindrance to their being enumerated' (Craig & Sinclair 2009: 116).

We cannot here enter into a debate about the merits of presentism, but we think this much can safely be said. Any theory of time that deserves serious consideration must make room for the possibility of enumerating things that are no longer (or not yet) present. Let's assume, at least for the sake of argument, that Craig theory passes this test. Then he is quite right to insist that a beginningless series of past events would be countably infinite. The bad news for him is that the same must be said of an endless series of future events. If, despite their non-existence, one can enumerate past events because reference and first order quantification are not ontologically committing, then we are equally entitled to enumerate things and events that do not yet exist but will do. Once this is granted, it is obvious (i) that the number of future events cannot be zero (unless we are at the very end of time!) and (ii) that the number of events in an endless series of events is identical to the number of events in a beginningless series of events. There just isn't any relevant difference between the two cases.

Craig seems to be grappling with this issue in the following passage.

> It might be said that at least there have been past events, and *so they can be numbered*. But by the same token there will be future events, so *why can they not be numbered?* Accordingly, one might be tempted to say that in an endless future there will be an actually infinite number of events, just as in a beginningless past there have been an actually infinite number of events. (Craig 2011: 116, emphasis added)

At this point, the reader naturally expects Craig to produce some reason for thinking that (non-existent) future events cannot be numbered, even though (equally non-existent) past ones can be; but he does nothing of the sort. (Indeed, how could he?) Instead, he merely circles back to the oft-repeated point that 'there never will be [will have been?] an actually infinite number of events since it is impossible to count to infinity' (Craig 2011, 116).

Here again Craig fails to distinguish between the *A(x)* function and the *B(x)* function. When he says that 'there never will be an actually infinite number of events', he must mean that there never *will have been* an actual infinite. This is a claim about the value of the *A(x)* function; it says that for no input of *A(x)* is the output infinite. This is true, but irrelevant. The issue concerns the number of events, each of which *will* happen, not the number of events that *will have* happened as time passes. That is, the issue concerns the value of the *B(x)* function, and the output of that function is certainly not zero! To the contrary, for *any* input of *B(x)*, the output is denumerable infinity.

IV. FUTURE EVENTS AS 'PURE POTENTIALITIES'

Craig also sometimes tries to establish a relevant asymmetry between past and future by contrasting the 'potentiality' of the former with the 'actuality' of the

latter. 'Future events', he says, 'have not as of yet been actualized, whereas past and present events have been actualized' (Craig 2011: 306). Unlike past events, therefore, future events are 'pure potentialities' and are 'not part of the actual world' (Craig 2010: 445–446).

Understood this way, Craig's account bears considerable resemblance to the growing block theory of time. The past may not 'exist', but it is nevertheless a continually growing block of actualised potentialities. The future, on the other hand, consists in potentialities that have not yet been actualised. In this way, Craig apparently thinks that he has provided a symmetry breaker that is relevant for our problem. That is to say, he thinks that the distinction between the actuality of the past and the potentiality of the future is the thing in virtue of which a beginningless past is a true actual infinite and an endless future is merely a potential infinite.

Notice, however, that this alleged asymmetry between past and future is completely irrelevant to the *number* of past and future events. Potentialities that will be actualised are in principle no less numerable than ones that have been. To see this, let us consider two simple counting scenarios featuring an imaginary being named 'Counter'.

Suppose first that:

> *Counter will begin his count one minute from now; one minute after that he will add one to his count; two minutes later, he will add one to his count; and after that he will never do any more counting.*

We can, if we like, think of this as a series of potential 'counting-events'. We can then ask how many distinct counting-events will – ever – be actualised. That is, we can ask for the number of potential counting-events, each of which will be actualised by Counter at some time or other. It would be absurd to say that the answer is anything other than three.

Consider next an 'endless count' scenario:

> *Counter will begin counting one minute from now; one minute after that and after every other future act of counting, he will add one to his count.*

Again, we can think of this as a series of 'pure potentialities', each of which will be actualised. And again, we can ask for the number of potential 'counting-events' that will – *at some time or other* – be actualised. It is easily proved by mathematical induction that for every positive integer n, Counter will actualise an n^{th} potential act of counting. Here, then, we have an *infinity of potentialities* – which (note well!) is *not* to be confused with *a potential infinity*.

V. BUT IS IT AN *ACTUAL* INFINITE?

At this point, some may be tempted to object that an infinity of not-yet-actualised potentialities can't be a true actual infinite. An actual infinite, they

may say, must be comprised of items that belong to what Craig calls 'the actual world' – comprised, that is, of items that are (or have been) actualised. So even if it is granted that an endless series of to-be-actualised potentialities is denumerably infinite, it might still seem that we have something like the asymmetrical result that has hitherto eluded us, viz. that a beginningless series of past events would be a true actual infinite (because it is comprised of infinitely many elements, each of which has been actualised), whereas an endless series of future ones would not be (because none of its elements has yet been actualised).

This won't do. In the first place, this manoeuvre is utterly question-begging. It says, in effect, that an endless series of future events can't be an actual infinite because they aren't (yet) present or past. We could – with at least equal legitimacy – settle matters the opposite way by stipulating a definition of an 'actual infinite' on which it is comprised of items that are or have been *or will be* actual. Indeed, we think we could do this with greater legitimacy. Recall that we are considering a series of future events, each of which *will be* present. Their grip on reality is therefore strong enough for any reasonable 'reality condition' that might be proposed for the elements of an actual infinite.

In the second place, and more importantly, we think it matters very little to our argument how one decides to use the term 'actual infinite'. What does matter is that, *with respect to the allegedly absurd-making implications of the actual infinite*, a beginningless series of events, each of which has occurred, and an endless series of events, each of which will occur, are in the same boat.

We have already shown that both have Cantor's property – that of having proper parts, the number of whose elements is equal to that of the whole. And we've seen that this is true *even given a dynamic theory of time*. As time passes, more and more potential counting-events are 'actualised'. They are, as it were, 'subtracted' from the series of yet-to-be actualised potentialities and added to the series of those that have been actualised. And yet, the number of past events is no greater than it was, and the number of future events is no smaller. Both are still denumerably infinite.

If we are to leave no stone unturned, however, we must also consider a worry about inverse arithmetic operations on transfinite numbers that figures prominently in the literature. This will require separate treatment.

VI. 'INFINITY MINUS INFINITY'

Imagine a collection of numbered items, one for each natural number, all existing in the present. If all the items numbered four or greater were removed, only three would remain. If all the odd-numbered items were removed, infinitely many would remain. And yet, the same number of items would have been 'subtracted' in both cases. Craig thinks this is a big problem. Indeed, he claims that inverse arithmetical operations are prohibited in transfinite

arithmetic because they 'lead to contradictions' (Craig & Sinclair 2009: 111). But 'in the real world,' he continues, we cannot prohibit the removal of items from a collection; and from this he infers that 'in the real world' (as opposed to the 'world' of mathematical cogitation) infinite collections are impossible.

We do not think that any 'real world impossibility' has been demonstrated. What's been shown is merely that in the case of an infinite collection, the remainder (after 'taking away' some of its members) depends, not just on the number of items 'taken away,' but on *which ones* are 'taken away'. Once these are identified, it is a perfectly straightforward matter to find a unique remainder and no contradiction emerges.

But our concern here lies in a different direction. Suppose (for the sake of argument) that there is some genuine problem about how many items remain when infinitely many have been 'removed' or 'taken away' from an infinite collection of objects. Two further questions must be addressed: what, if anything, does this tell us about the possibility of a beginningless series of events? and does it tell us the same thing about an endless one?

With respect to the first question, one might have thought that Craig's argument simply does not apply to a series of past events. Once something has happened, it will forever be the case that it has happened. It follows that nothing is ever 'removed from' or 'subtracted from' the past. So even if the past were infinite, there could not be a pair of 'real world' situations such that: (a) in both situations infinitely many items have been 'subtracted' from the past, (b) in one case finitely many items are 'left over', and (c) in the other case infinitely many are 'left over'.

Craig is unmoved by this objection. He insists that:

> . . . we can still compare, for example, the number of odd-numbered events with the total number of events, or the number of events prior to today with the number of events prior to any point in the past, and *mentally add and subtract* such events so as to obtain the same absurdities' (Craig 1999: 64, emphasis added).

Following a suggestion by J. P. Moreland (2003), we can spell out Craig's idea in the following way. The past could have been different; so one can 'mentally subtract' past events by imagining counterfactual possibilities in which they never occurred. If we do this for all but (say) the last three of those events, we will have mentally subtracted infinitely many and will have a 'remainder' of three. If, on the other hand, we mentally subtract (say) all the odd-numbered ones, we will again have mentally subtracted infinitely many, but this time with a countably infinite 'remainder' (the even-numbered ones).

Fair enough. But now it is perfectly obvious that a parallel thought experiment yields precisely the same result for an endless series of future events. We can 'mentally subtract' events from the future – from the series of potentialities that will be actualised – by imagining counterfactual possibilities in which they won't occur. If we 'mentally subtract' all but the first three items in an endless

series, we will have subtracted infinitely many with a 'remainder' of three. But we can also 'mentally subtract' *every other one* of them (all the odd-numbered ones, say), in which case we will again have subtracted infinitely many, this time with a 'remainder' that is countably infinite.

Once again, therefore, the parallel between past and future holds up. So if (unlike us) you think that Craig's 'infinity minus infinity' argument shows that a beginningless series is impossible, then you should say the same of an endless series.

VII. WHAT IF AN ACTUAL INFINITY OF THINGS COULD NOT EXIST IN THE PRESENT?

At this point, some proponents of the Kalam argument may concede that an endless series of events is both possible and a true actual infinite. It is not the actual infinite per se, they may say, that generates the allegedly absurd implications of (say) an infinite hotel, but rather an actual infinity of things *existing at the same time*. Even so, it might be argued that a beginningless past is impossible, because (unlike an endless future) it *could have given rise to* an infinite collection of objects existing together in the present, and because anything that entails the possibility of an impossibility must itself be impossible.

An apparent benefit of this approach, for a defender of the Kalam argument, is that it seems to deliver a clear symmetry breaker whilst sidestepping altogether the issues discussed above relating to the ontology of past and future events. There is no need to claim that future events don't exist, or to deny that they are a countable infinity. It is not even necessary to endorse an A–theory of time. If the argument is sound, it works just as well on the B–theory.

There are two distinct issues here: (i) is it the case that an infinite collection of objects existing together in the present is impossible? and (ii) does the possibility of an infinite past entail the possibility of such a collection? The example most often discussed in connection with (i) is that of an infinite hotel – known in the literature as a Hilbert's Hotel (hereafter, a HH). Let's assume (for the sake of argument) that a HH is as impossible as some people say and that this warrants an affirmative answer to question (i). That leaves us with question (ii). Does the possibility of a beginningless past bring with it the possibility of (say) a HH?

An argument for saying that it does has been put forward by Andrew Loke.[4] He writes:

> Suppose this is how Hilbert's Hotel is constructed: there exists a 'hotel room builder' who has been building hotel rooms at regular time intervals as long as time exists. Suppose there also exists a 'customer generator' which has been generating customers who checked in the hotel at regular time intervals as long as time exists. Suppose that

[4] A similar argument can be found in Benerdete (1964), p. 270, and Alexander Pruss (2009).

> the hotel rooms and the customers continue existing after they have been built and generated, respectively. Now if the actual world is one in which the universe is past-eternal, then there would have been an actual infinite number of time intervals, and an actual infinite number of hotel rooms and customers occupying the rooms. In other words, if the actual world were one in which the universe is past-eternal, then there would be a world in which an actually infinite number of things have been actualized (Loke 2014, p. 49).

Loke's central idea is that if time had no beginning then an infinite collection of co-existent items could have been generated by way of beginningless, step-by-step process. So let's focus on this claim. In doing so, it will be convenient to simplify the example a bit. Let's focus just on the creation of an infinite hotel. (For ease of exposition, we can set the guests to one side.) Let the 'builder' be God; and imagine that, at regular intervals throughout the whole of time past, God creates new hotel rooms ex nihilo, and guarantees their persistence. If time has no beginning, then an actual infinity of hotel rooms must exist in the present. Thus, the infinitude of the past seems to entail that there could have been an infinity of co-existent items.[5]

But the same cannot be said of the future, one might think. After all, if God starts making hotel rooms now, and continues to do so for the rest of time, and even if the future has no end, there will only ever be a finite hotel in existence at any particular moment. Thus, the endless future does not have the same entailments as the infinite past; specifically, the infinite past entails that a HH is possible, whereas the infinite future does not entail this. If we can assume that a HH is metaphysically impossible in the present, we would be able to conclude that the infinite past is metaphysically impossible in a way that would not also apply to the infinite future, and we would have distinguished between past and future in a way that is friendly to the Kalam argument.

We can state Loke's argument this way:

1. If time had no beginning, then a HH would be metaphysically possible.
2. A HH is metaphysically impossible.
3. Therefore, time has a beginning.[6]

Many will be tempted by arguments like this one. Yet it also has a subtle flaw, which we shall explain. Key to understanding this is the relationship between the impossibility of HHs (which we are assuming for the sake of argument) and God's omnipotence.

If an omnipotent God had completely unrestricted power, then he would have the ability to make a HH appear all in one go. Indeed, supposing Loke's

[5] Alexander Pruss (2009) puts Loke's argument this way: 'If there could be a backwards infinite sequence of events, there could be a backwards infinite sequence of events during each of which a hotel room is created, none of which are destroyed. An infinite number of hotel rooms would then be the result'.

[6] In personal correspondence, he has indicated that this is his argument.

hotel room builder was bound to make hotel rooms at regular intervals for the rest of an endless future, then (as Yishai Cohen notes) God could simply decree 'let every hotel room that will be built at some time later come into being now', and thus instantaneously bring about an actual infinite number of hotel rooms (Cohen 2019: 293). Cohen's conclusion is that the existence of an omnipotent being such as this thereby blocks Loke's scenario from providing a relevant symmetry breaker. A theist who believes in such a being cannot thereby appeal to Loke's argument to provide the required symmetry breaker.

There is a natural way for a defender of the Kalam to respond to this, which is to insist that omnipotence doesn't mean that God can do literally *anything*. Almost everyone agrees that God cannot do that which is logically impossible, for instance. Thus, the question is not whether God's omnipotence has any restrictions, but *which* restrictions it has. One plausible suggestion is that God has the ability do anything metaphysically possible, but nothing metaphysically impossible. And if we take this route, then we have a reply to Cohen at hand: God cannot simply click his fingers and bring about the existence of a HH now, because the existence of a HH is metaphysically impossible. Such a restriction would imply that were God to say 'let every hotel room that will be built at some time later come into being now', and if there is an infinite number of hotel rooms such that each will be built at some time in the future, then God's decree would simply fail to have any effect. It would be beyond God's abilities to make such a thing happen. It would be no more effective than if he said 'let there be a triangle with four sides'. Thus, Cohen's objection only works by presupposing an unrealistic, and extreme, form of God's omnipotence; when omnipotence is restricted in the right way, the objection is defused.

However, the defender of the Kalam who makes this move is not out of trouble. Someone who responds to Cohen in this manner has very specific commitments which we can bring out by reference to a scope distinction captured in the following two formulas:

A) $\forall n \Diamond$(God made n hotel rooms)
B) $\Diamond \forall n$(God made n hotel rooms)

Formula A says that for *each* natural number n, it is possible that God made a hotel with n many rooms in total. Formula B says that it is possible that God made a hotel so big that there is a hotel room for *every* natural number n.

Given that Cohen's opponent is committed to both the metaphysical impossibility of HHs, and the restricted omnipotence of God, it follows that they consider formula A to be true, and formula B to be false. They will say that God *could* make a hotel all in one go such that the hotel rooms are numbered 1 to n, for any n (formula A), but he *could not* make a hotel all in one go where every single natural number corresponds to a unique room (formula B). We can put it in a slogan like this: *each is possible, but not all*.

Our objection runs as follows. Even if there is an infinite number of past events such that at each of them God could have made a persistent hotel room, so long as his omnipotence is restricted, it doesn't follow that a HH is metaphysically possible. A hotel room builder with restricted omnipotence has the ability to spend *any finite number* of those past times creating hotel rooms (formula A), but not to devote *every one* of an infinite number of past times to creating hotel rooms (formula B). That is, a builder with restricted omnipotence could make it the case that the totality of past room-creation-events is numbered from 1 to *n*, for any *n*, but not that there is a unique past room-creation-event for every *n*. To insist that the builder have the ability to do the latter is precisely to insist on violating the restriction on omnipotence that the defender of the Kalam argument just appealed to in order to respond to Cohen.

Many may see these cases as relevantly different however, in such a way that the restriction does not apply to the two cases in the same way. The relevant difference they may appeal to would be the instantaneousness of the Cohen scenario, compared with the diachronic nature of the Loke scenario. In private correspondence, Loke expressed a version of this thought as follows:

> 'God cannot build a HH at one go because the metaphysical impossibility of a HH would act as a constraint to prevent Him from doing that. However, can God build one room at a time? Sure He can. There is nothing that could prevent God from building a persistent room at t_{-1}, and nothing could prevent God from building a persistent room at t_{-2}, . . . Etc.'

And:

> '. . . the two examples [creating a HH instantly, and over time] are not equivalent in a crucial way: one involves 'at one go' whereas the other involves a process. This makes a difference because to prevent someone from doing something the prevention has to happen before or at the instant of action. However, after the person has acted, he can't be prevented from acting because what has been done has already been done. What this implies is that later conditions cannot prevent earlier events from happening.'[7]

In the case of creating a HH all in one go, if God said 'let there be a HH', he would find that his decree would fail to have any effect. But in contrast, if God were creating hotel rooms one at a time, and on each occasion saying 'let there be a single additional hotel room', etc, then each time he would find that his utterance is effective. This establishes a difference between the two cases. One is a single infinite step (which the omnipotence restriction forbids), and the second is an infinite number of finite steps (each of which the omnipotence restriction allows).

Our objection was that (given the impossibility of a HH) he cannot take *all* of the little individual steps. But, intuitively, there is nothing that could

[7] Quoted with permission.

ever prevent him from taking any additional step. We could perhaps say that at some point in the past God *must* have stopped making hotel rooms. But why think this? We cannot simply say that the impossibility of a HH at t_0 guarantees that he stops making hotel rooms at any earlier time; as Loke says 'later conditions cannot prevent earlier events from happening'. Here is the dilemma: if he never stopped, it looks like there would be a HH now; but any insistence on there being a stopping point in the past looks unacceptably ad hoc, and the sheer impossibility of a HH at t_0 can't influence prior actions. This seems to be Loke's objection.

We have two responses to this objection. First of all, there is an analogy again with the Cohen case, which is just a spatialization of the temporal case. Imagine an infinitely long street, 'Infinite Avenue', with addresses '1 Infinite Ave', '2 Infinite Ave', etc. God has the ability to successfully decree, for instance, 'let there be a hotel at 'n Infinite Avenue'', for any *n*. Thus, the two cases look directly analogous:

- There is no address on Infinite Avenue that if God decreed that a hotel should exist there, that decree would be unsuccessful. Yet, he cannot make a hotel at every address on the avenue.
- There is no time in the infinite past that if God had decreed that a hotel room should begin to exist then, that decree would have been unsuccessful. Yet, he cannot have made a hotel room at every time in the infinite past.

If anything is ad hoc, it seems to us that it is the insistence the restriction is acceptable in the spatial case but not the temporal one. They seem entirely analogous to us. We would want to see a symmetry breaker for this, before we can accept that the argument itself is a symmetry breaker between the beginningless past and endless future.

However, it may seem that we are missing Loke's point, which was about 'prevention'; how can a future stage influence a past one? For those who have this thought, we offer our second objection. What 'prevents' God from adding hotel rooms one at a time and arriving at a HH is that *this is already a metaphysically impossible situation*. Loke's suggestion, which painted the picture as if it required some spooky backwards in time influence from later conditions to earlier events, is in fact a straw man. If at t_{-1} God creates a single room, and then later at t_0 there is a HH, it follows that there *already was* a HH at t_{-1}. After all, the addition of one hotel room (or any finite number of hotel rooms) cannot ever be enough to change a finite hotel into an infinite one. One cannot, as it were, traverse an actual infinite in finite steps. So, if there is a HH present now, then at every previous time, whenever God was adding hotel rooms, he was adding them to a HH. But adding a room to a HH is metaphysically impossible, just because the existence of a HH is metaphysically impossible! Generally, from $\sim\Diamond p$, and $\Diamond q$, it follows that $\sim\Diamond(p \,\&\, q)$. Thus, the mistaken presupposition in Loke's response here is that we ever need to appeal to later

conditions in order to rule out the earlier events in this scenario; if there is a HH at any time, then each previous room building event is co-present with a HH. At least, that's how it is so long as God's omnipotence is restricted. If he were able to make a HH all in one go, then he could have added hotel rooms to a hotel in the past that was finite, but which is infinite now. But as we saw, that option has been ruled out to get past Cohen's objection. Yet, that very restriction here puts the defender of the Kalam right back into trouble again.

Thus, we think that arguments like Loke's, which attempt to fashion a symmetry breaker by appealing to the fact that a beginningless past would entail the possibility of a HH in the present are also misguided. In order to avoid Cohen's objection, a restriction on God's omnipotence seemed to be required. But that very restriction makes the scenario whereby God makes a HH by successive addition a metaphysically impossible situation – one which in particular violates the notion of prevention Loke appeals to.

VIII. CONCLUDING REMARKS

We have established two things. First, that as far as the actual infinite is concerned, a beginningless series of past events and an endless series of future ones are in the same boat. We think (and we expect most friends of the Kalam argument to agree) that an endless series of events, each of which will occur, is at least metaphysically possible. But then an actual infinity of events occurring one-after-another is plainly possible, in which case the possibility a beginningless series of past events should not be rejected *merely* on the ground that it would be an actual infinite.

Second, we have shown that the possibility of an infinite past does not stand or fall with the possibility of an actual infinity of presently existing things. So even if one thinks that (say) a Hilbert's Hotel is impossible, it does not follow that the past must have a beginning.

There are, of course, other arguments for the finitude of the past that we have not discussed – most notably, perhaps, the one based on the supposed impossibility of 'traversing the infinite'. We shall have to leave them for another occasion.

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